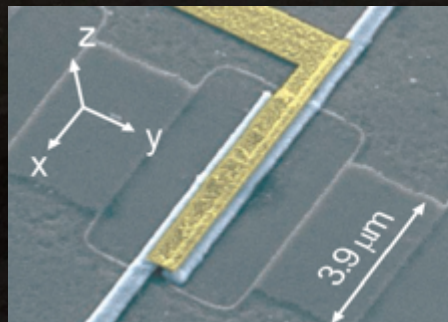


Metal-Insulator Transition and Beyond in the Pyrochlore Iridates

Leon Balents, KITP



William I. Fine
Theoretical Physics Institute

UNIVERSITY OF MINNESOTA

**SYMMETRIES & INTERACTIONS
IN TOPOLOGICAL MATTER**

May 1 - 3, 2015

People



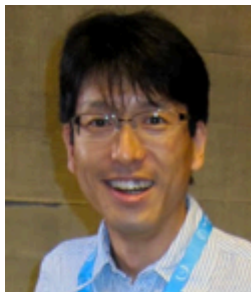
Lucile Savary



Ru Chen



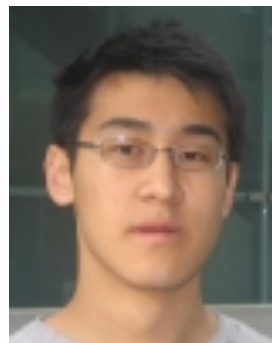
Eun-Gook Moon



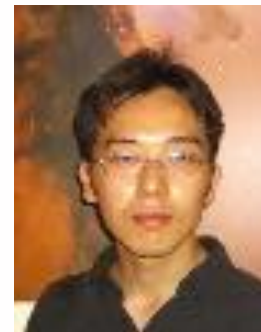
S. Nakatsuji



T. Kondo

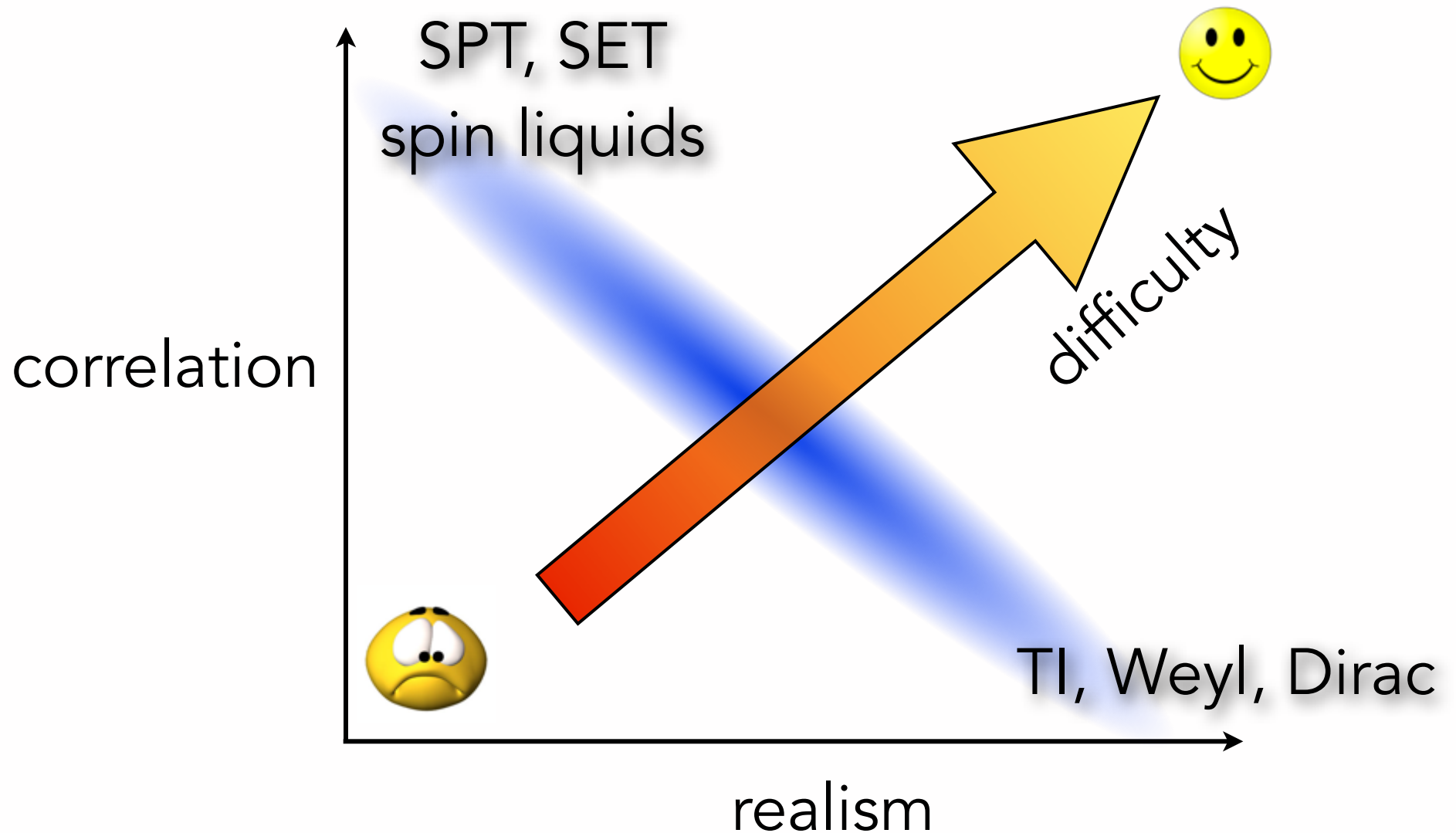


T. Hsieh



H. Ishizuka

Theory Space



Weyl semimetal

937

PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING

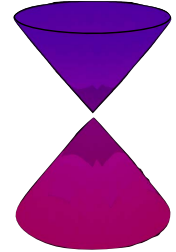
Princeton University, Princeton, New Jersey

(Received June 16, 1937)



For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neighborhood of a point \mathbf{k} where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa \rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

$$H = v \vec{\sigma} \cdot \vec{k}$$



A two-component spinor in three dimensions: "half" of a Dirac fermion. Weyl fermions have a chirality and *must* be massless

Weyl semimetal

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neighborhood of a point \mathbf{k} where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa \rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

937

PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

CONYERS HERRING

Princeton University, Princeton, New Jersey

(Received June 16, 1937)

Either inversion or time-reversal
(or both) must be broken

Where it “started”

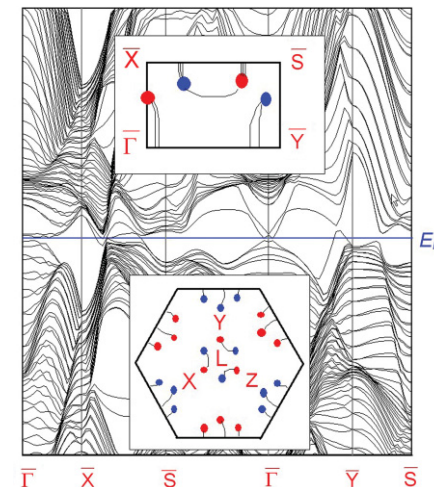
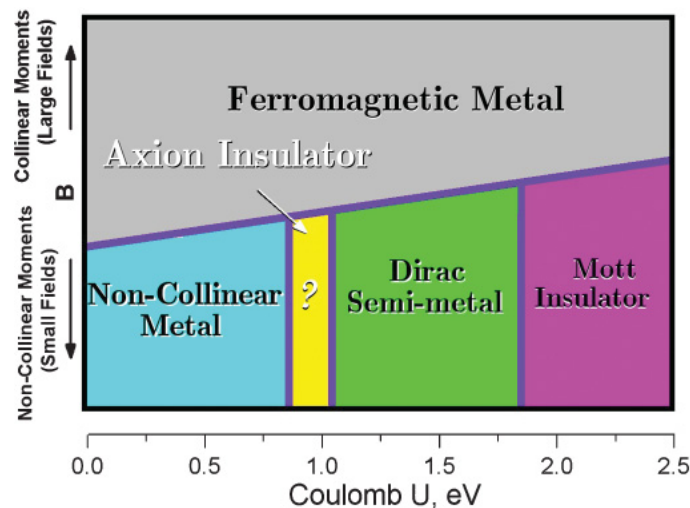
PHYSICAL REVIEW B **83**, 205101 (2011)



Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates

Xiangang Wan,¹ Ari M. Turner,² Ashvin Vishwanath,^{2,3} and Sergey Y. Savrasov^{1,4}

- Predicted Weyl semimetal in $\text{Y}_2\text{Ir}_2\text{O}_7$ pyrochlore iridate



c.f. Burkov Balents 2011

Where it “started”

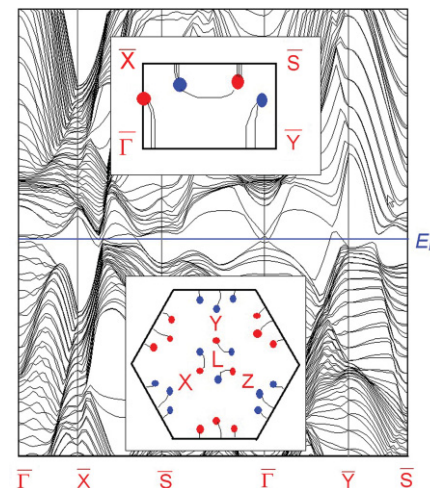
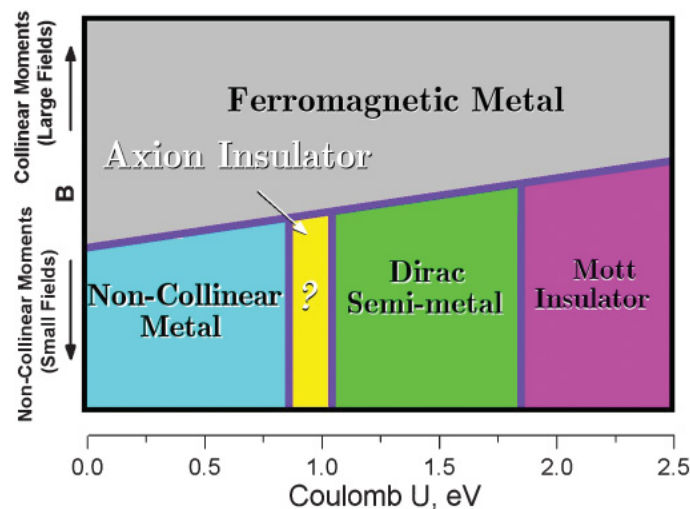
PHYSICAL REVIEW B **83**, 205101 (2011)



Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates

Xiangang Wan,¹ Ari M. Turner,² Ashvin Vishwanath,^{2,3} and Sergey Y. Savrasov^{1,4}

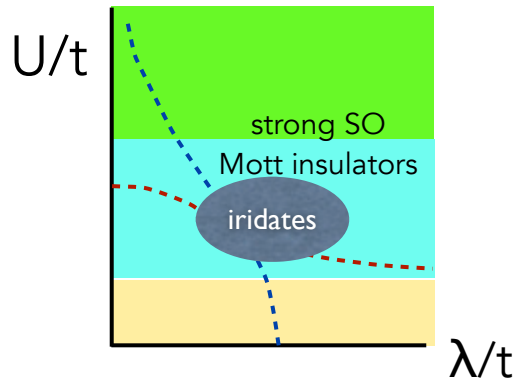
- Predicted Weyl semimetal in $\text{Y}_2\text{Ir}_2\text{O}_7$ pyrochlore iridate



What happened to this thing?

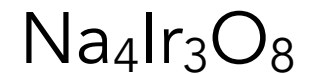
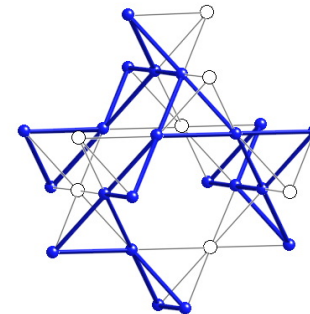
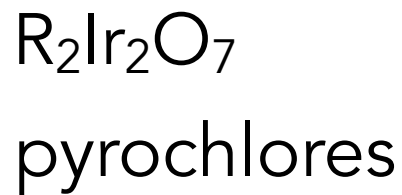
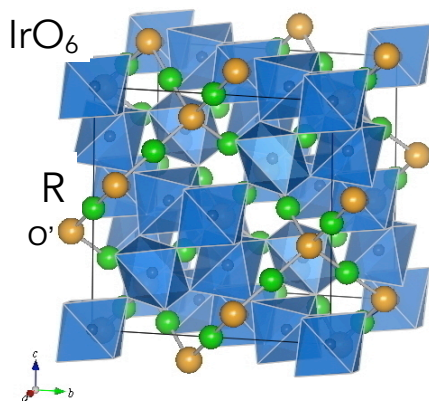
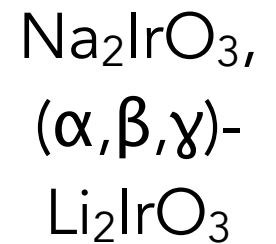
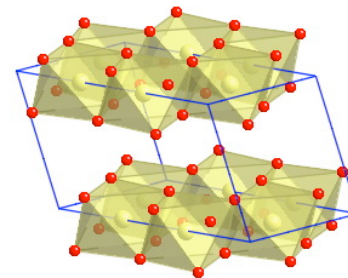
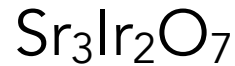
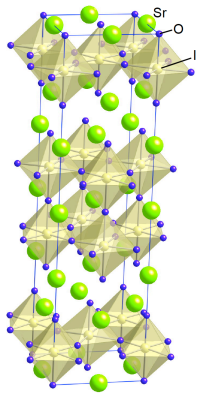
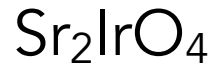
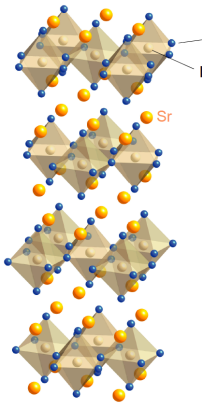
Plan

- Review pyrochlore iridates.
- “Clean” theory
- Less clean reality. Weyl not?



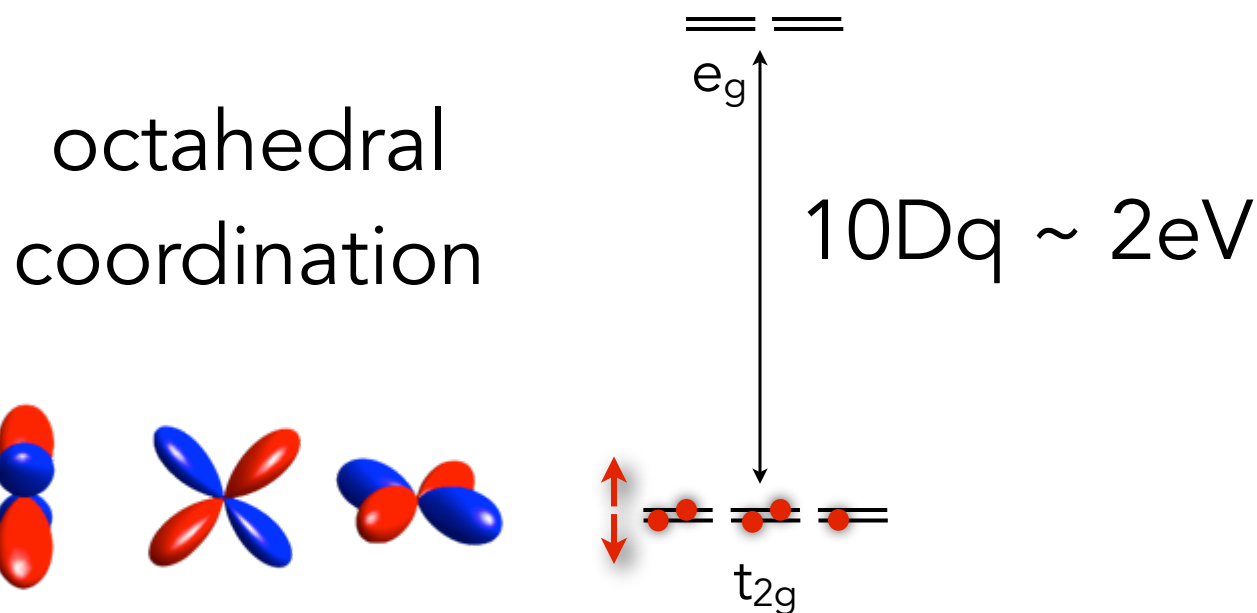
Iridates

- All energy scales comparable $U \sim W \sim \lambda$



Spin-Orbit Coupling

$$\text{Ir}^{4+} = 5d^5$$

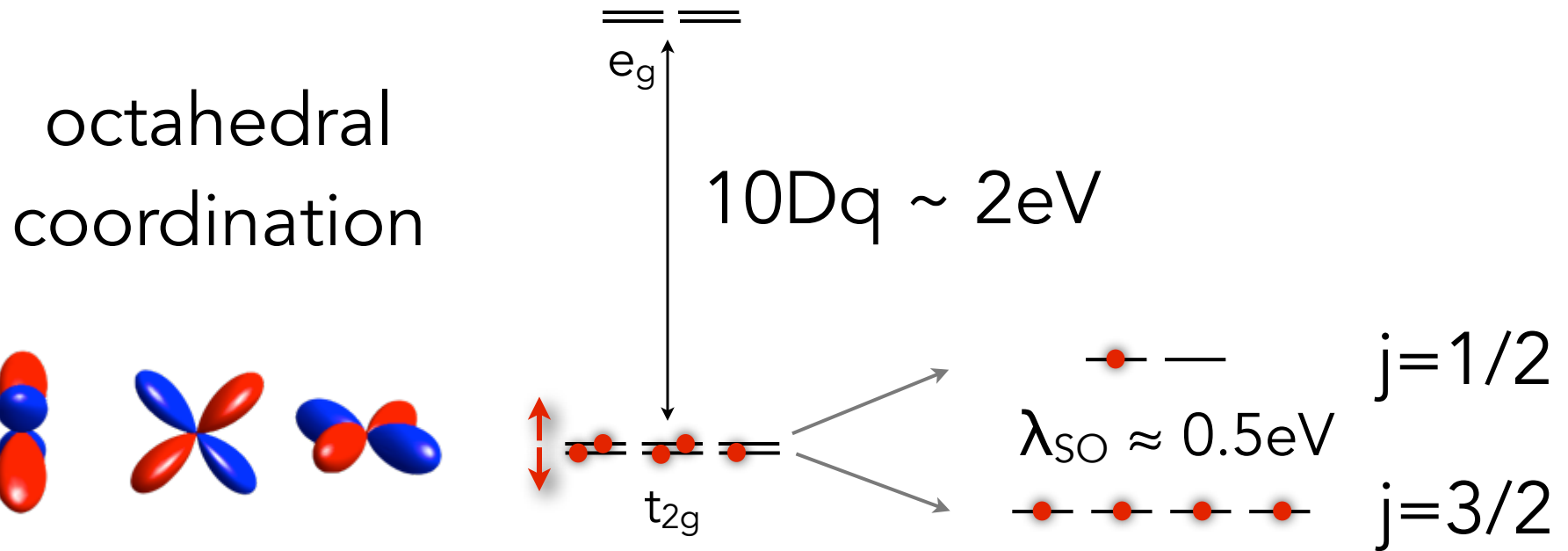
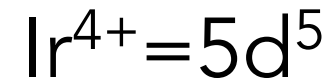


$$P_{t_{2g}} \mathbf{L} P_{t_{2g}} = -\mathbf{L}_{\text{eff}}$$

$$\ell_{\text{eff}} = 1$$

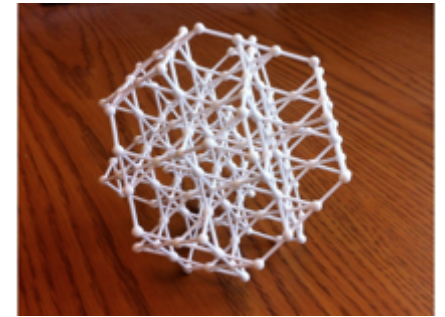
$$H_{\text{SOC}} = -\lambda \mathbf{L}_{\text{eff}} \cdot \mathbf{S}$$

Spin-Orbit Coupling

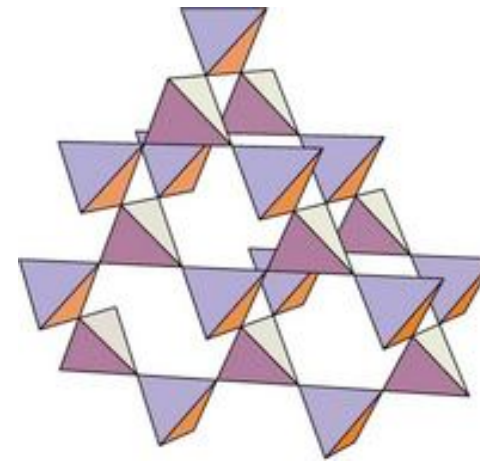
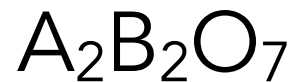
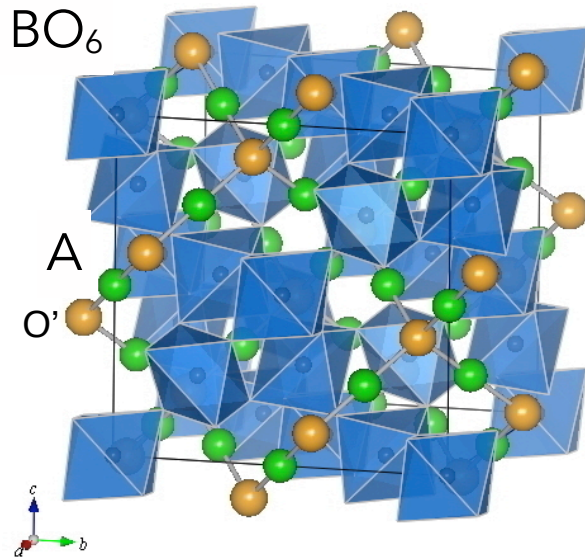


Note: only Kramer's degeneracy remains. SOC tends to suppress Jahn-Teller effects.

Pyrochlores



<http://www.shapeways.com/shops/cmm.html>

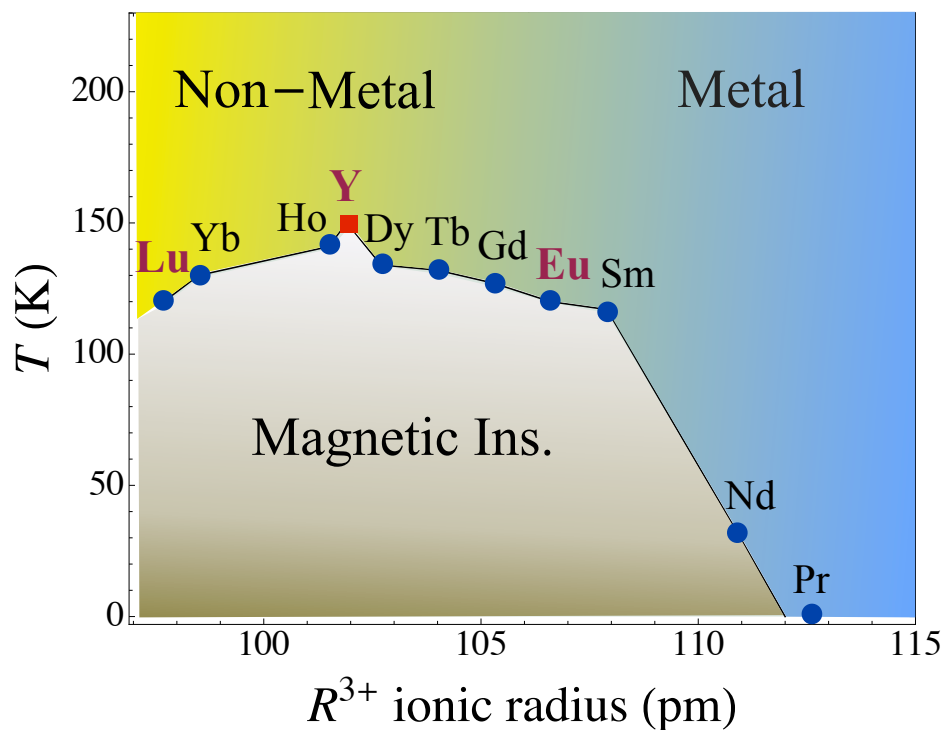


A (B) sublattice

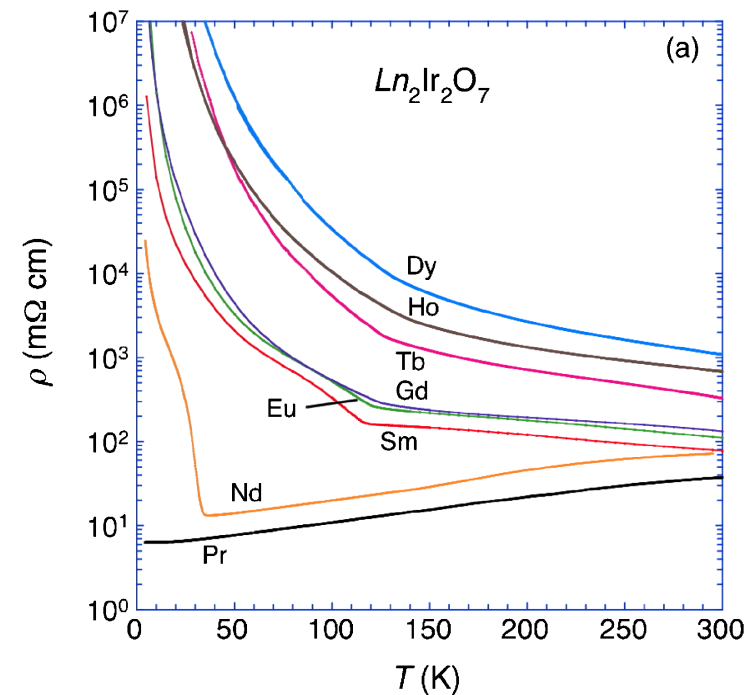
$A = Y, Ln$ ← f-electron moments, except
 $B = Ir$ when $A = Y, Eu$

Pyrochlore iridates

- Continuous magnetic/metal-insulator transitions

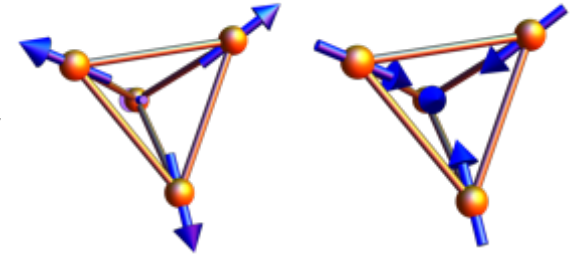


Yanagashima+Maeno, JPSJ 2001
 K. Matsuhira et al, JPSJ 2011
 W. Witczak-Krempa et al, ARCMP 2013

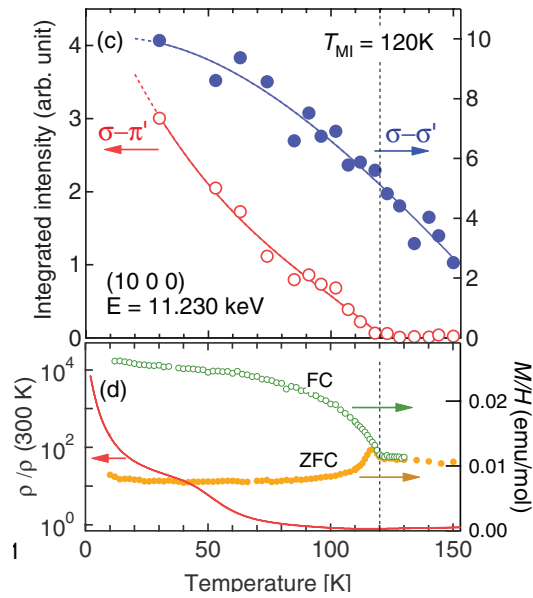


K. Matsuhira et al, JPSJ 2011

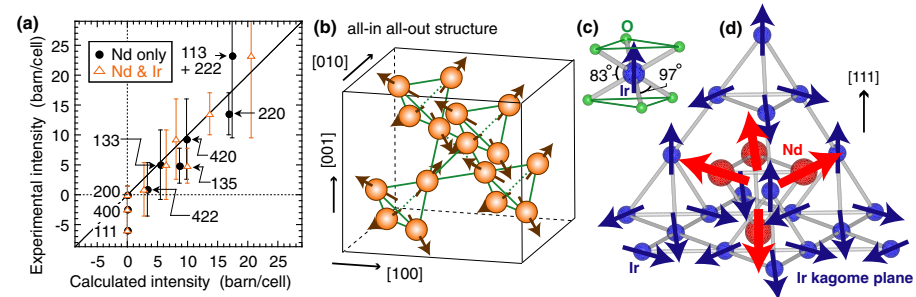
AIAO order



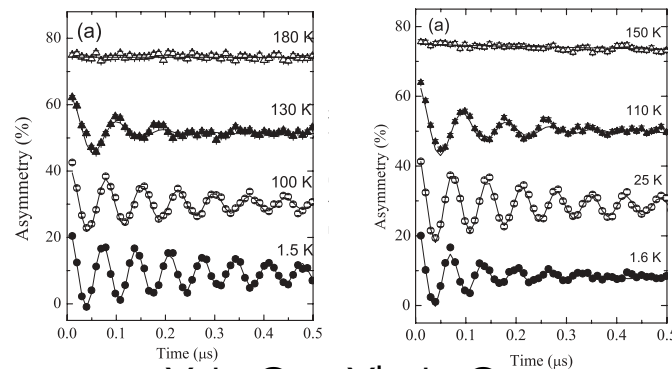
- Growing evidence in many materials



$\text{Eu}_2\text{Ir}_2\text{O}_7$
H. Sagayama
et al, 2013



$\text{Nd}_2\text{Ir}_2\text{O}_7$
K. Tomiyasu *et al*, 2012

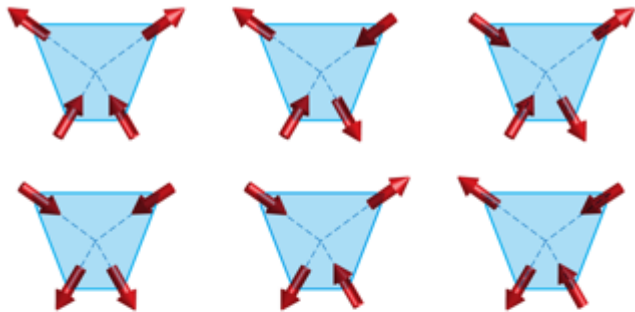
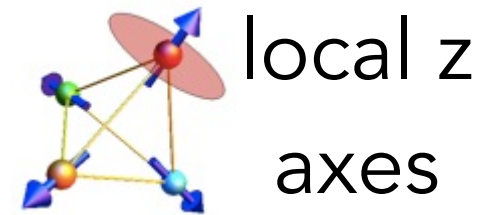


$\text{Y}_2\text{Ir}_2\text{O}_7$, $\text{Yb}_2\text{Ir}_2\text{O}_7$
S. Disseler *et al*, 2012

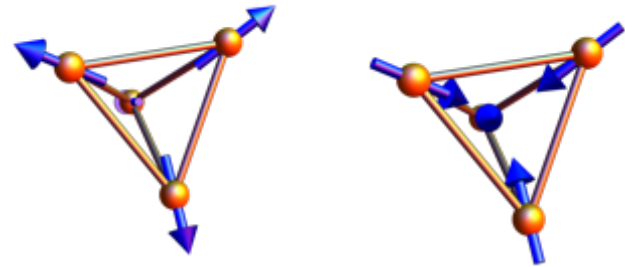
Local moment picture

- Ising model

$$H = J_z \sum_{\langle i,j \rangle} S_i^z S_j^z$$



$J_z > 0$: spin ice

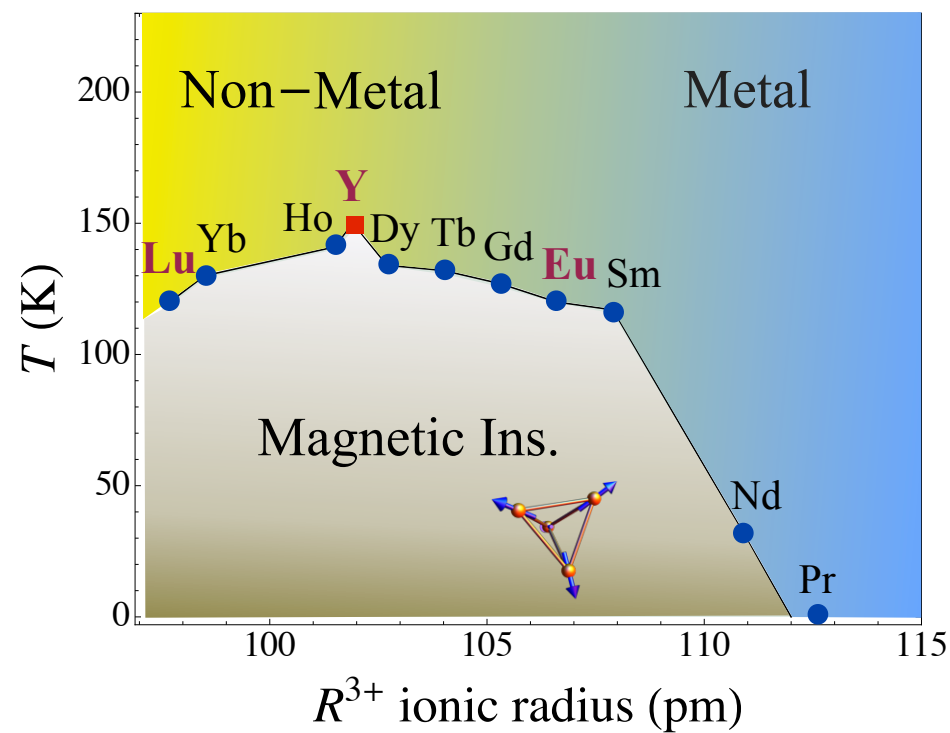


$J_z < 0$: all-in/all-out order



Quantum criticality

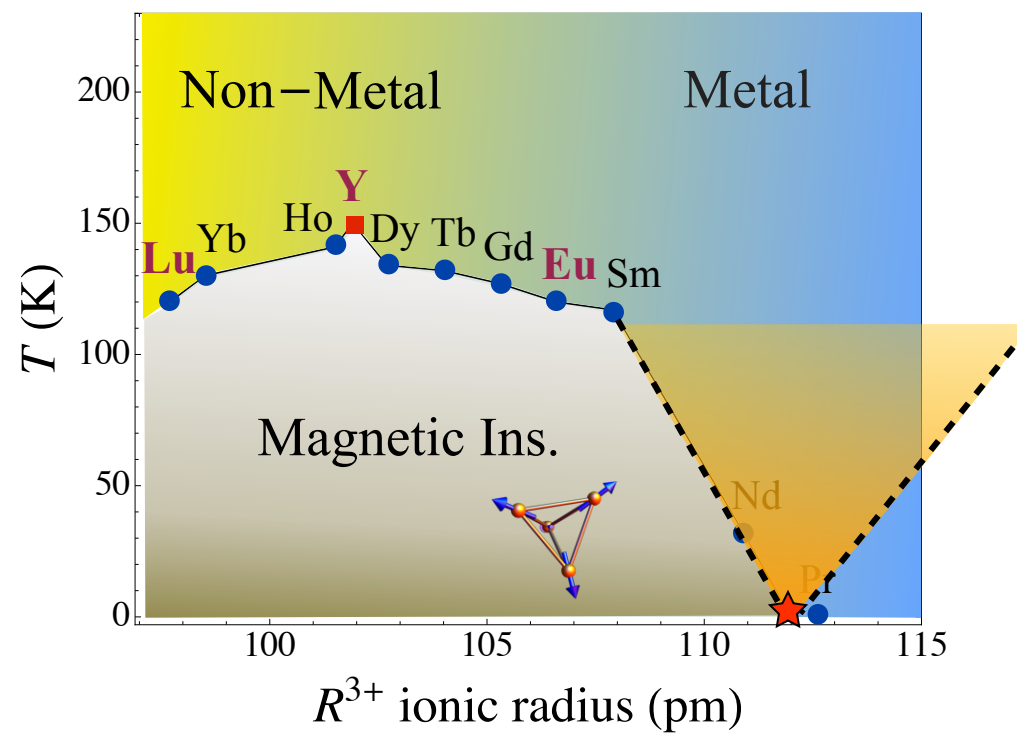
How does magnetic order onset?





Quantum criticality

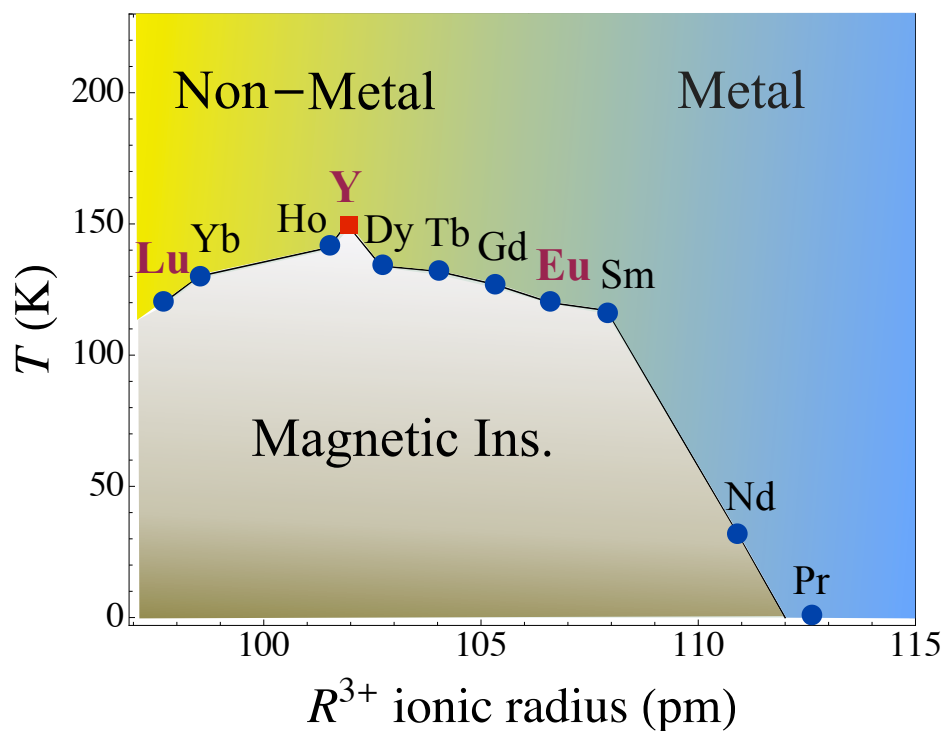
How does magnetic order onset?



QCP?

Pyrochlore iridates

- Continuous magnetic/metal-insulator transitions

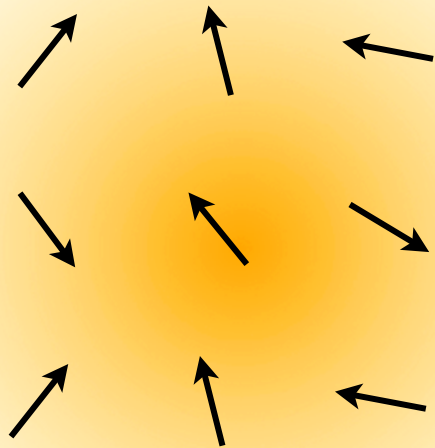


Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCMP 2013

note: Y, Eu fit on the same plot. Indicates f electron spins play a secondary role

c.f. $J_{\text{spin ice}} \sim 1\text{K}$

Energy scales

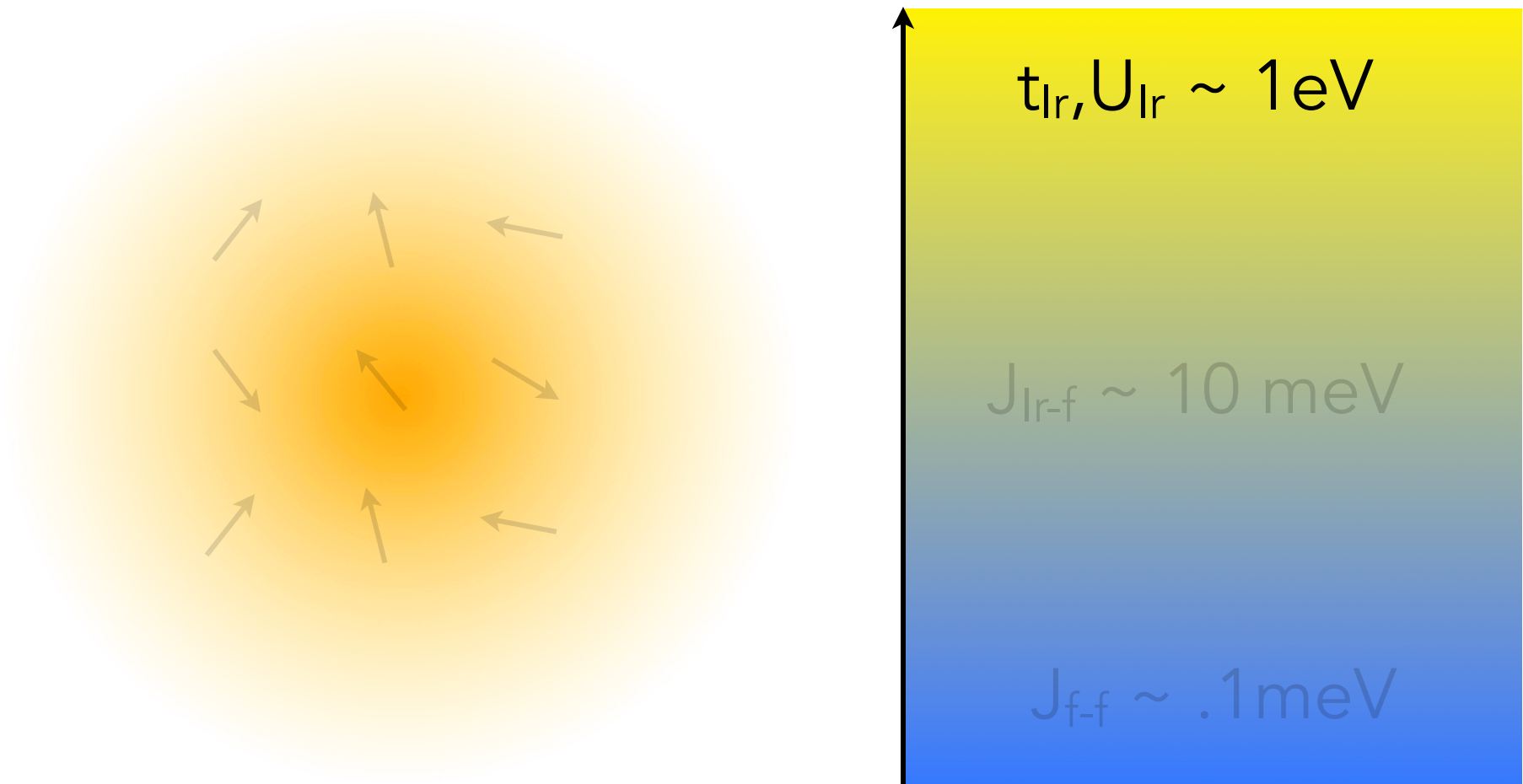


$$t_{lr}, U_{lr} \sim 1\text{eV}$$

$$J_{lr-f} \sim 10\text{ meV}$$

$$J_{f-f} \sim .1\text{meV}$$

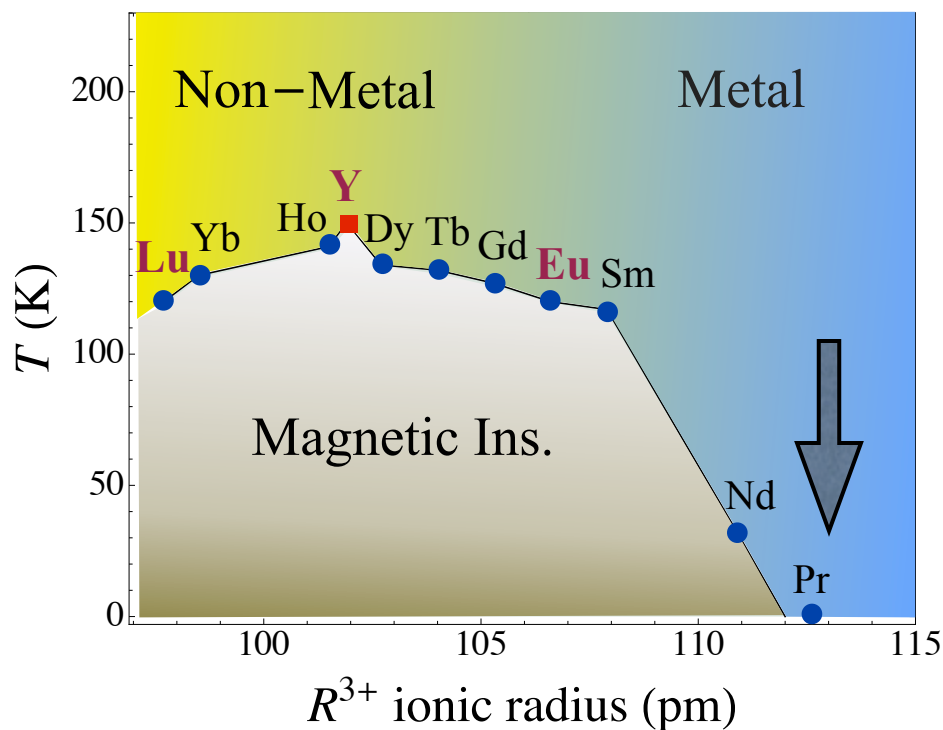
Energy scales



Neglect rare earths at first cut

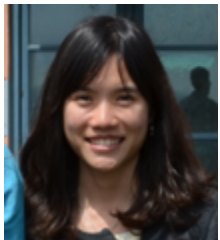
Pyrochlore iridates

- Itinerancy?



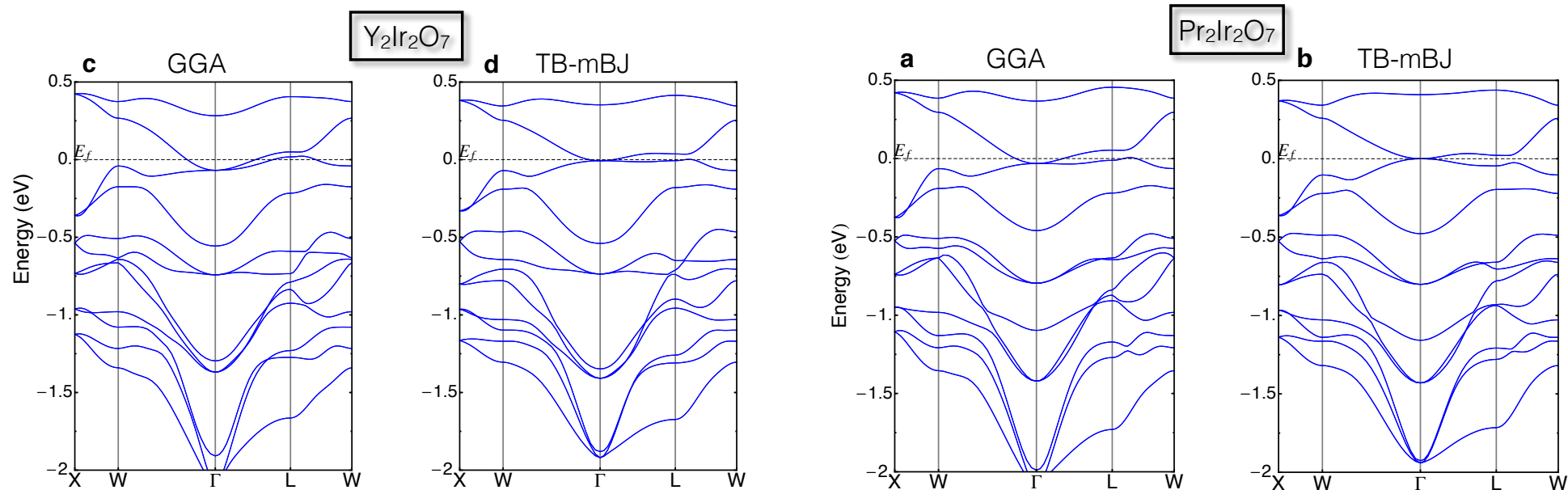
Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCOMP 2013

what is the “parent”
paramagnetic
structure?

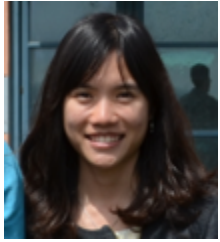


paramagnetic, GGA+SO, Wien2k

Electronic structure

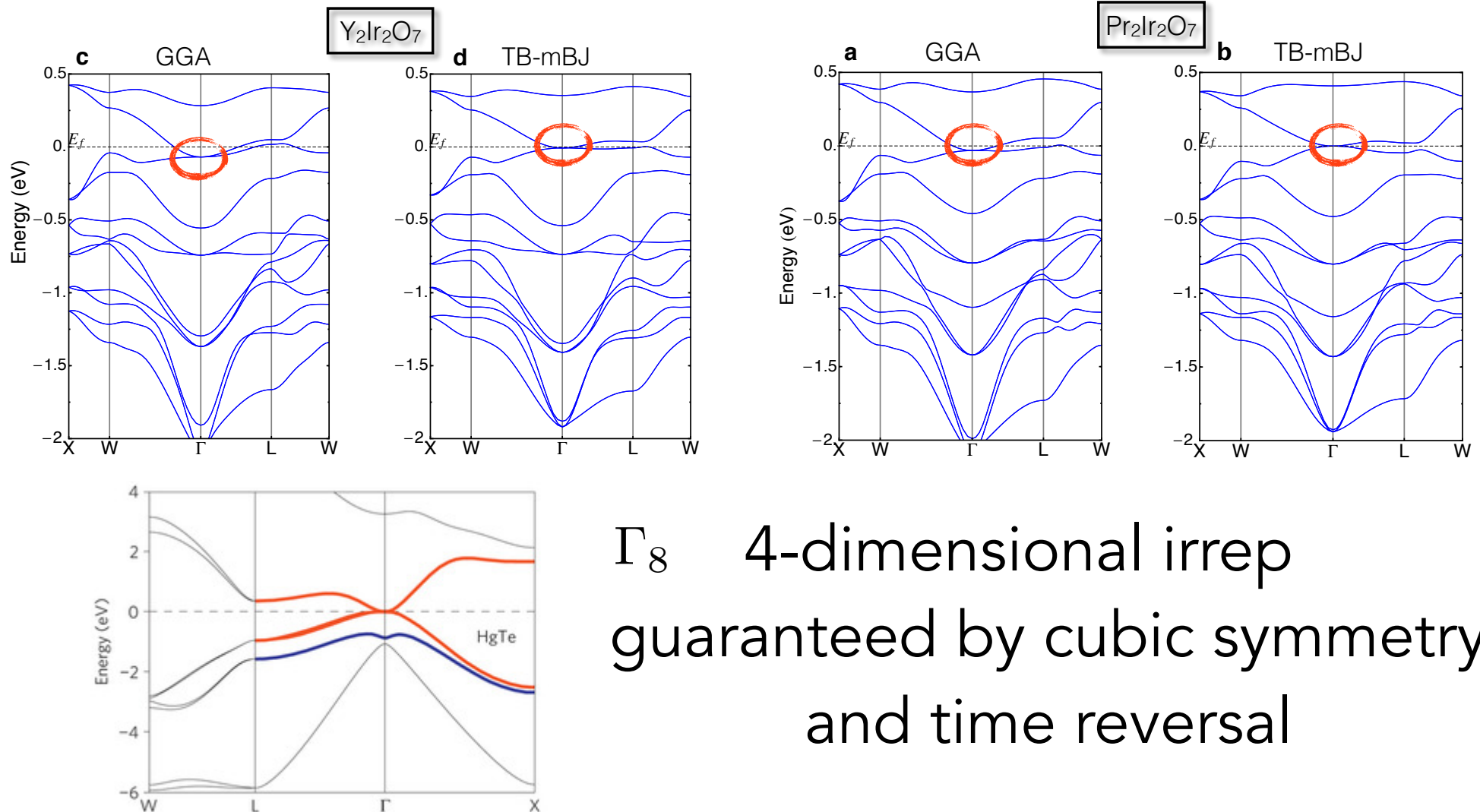


primarily Ir 5d bands

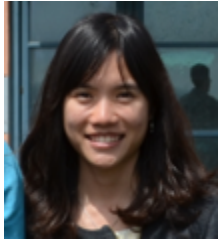


paramagnetic, GGA+SO, Wien2k

Electronic structure

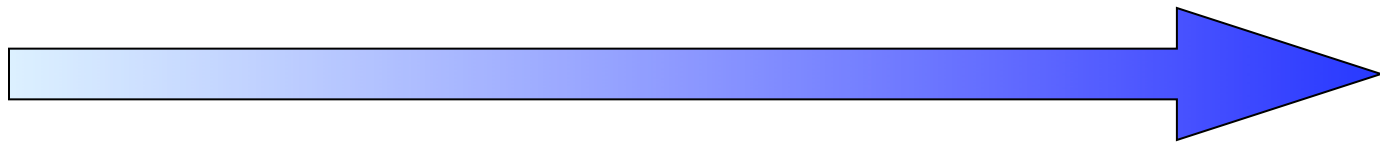
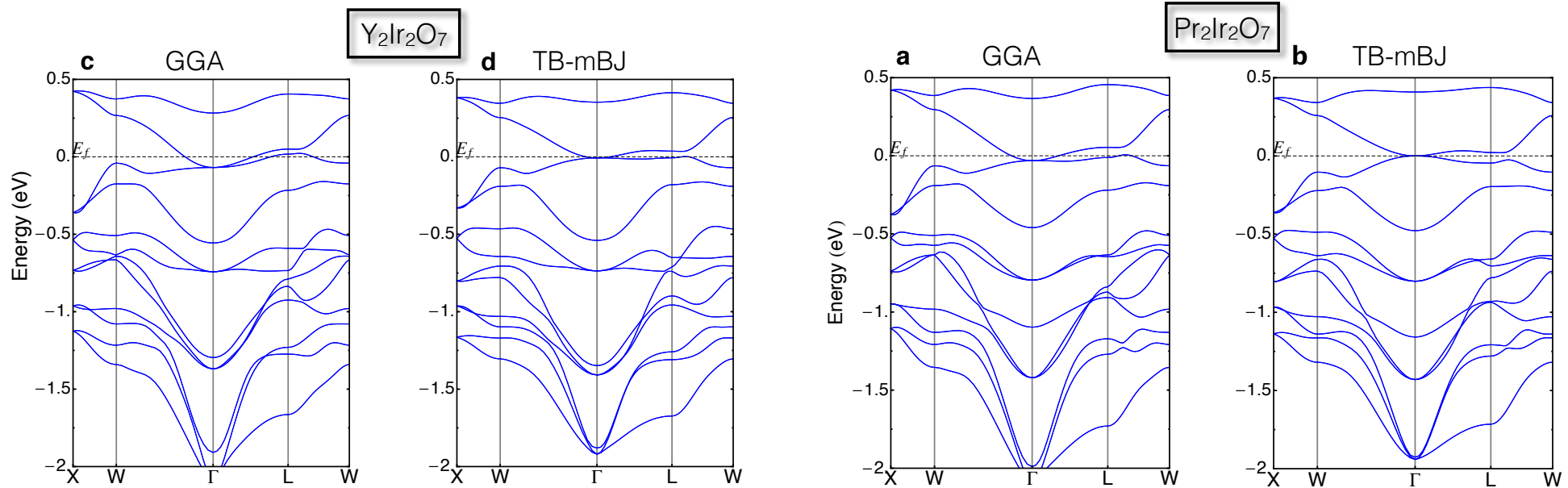


Γ_8 4-dimensional irrep
guaranteed by cubic symmetry
and time reversal

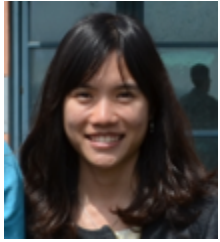


paramagnetic, GGA+SO, Wien2k

Electronic structure

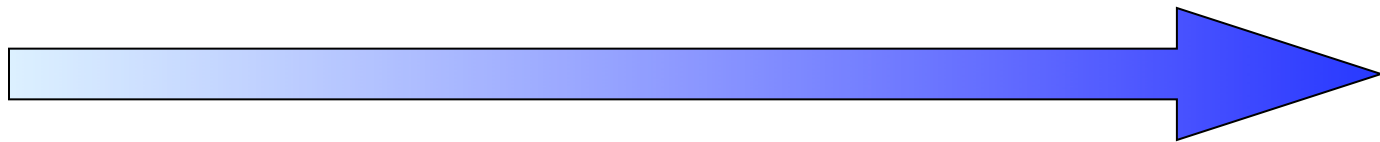
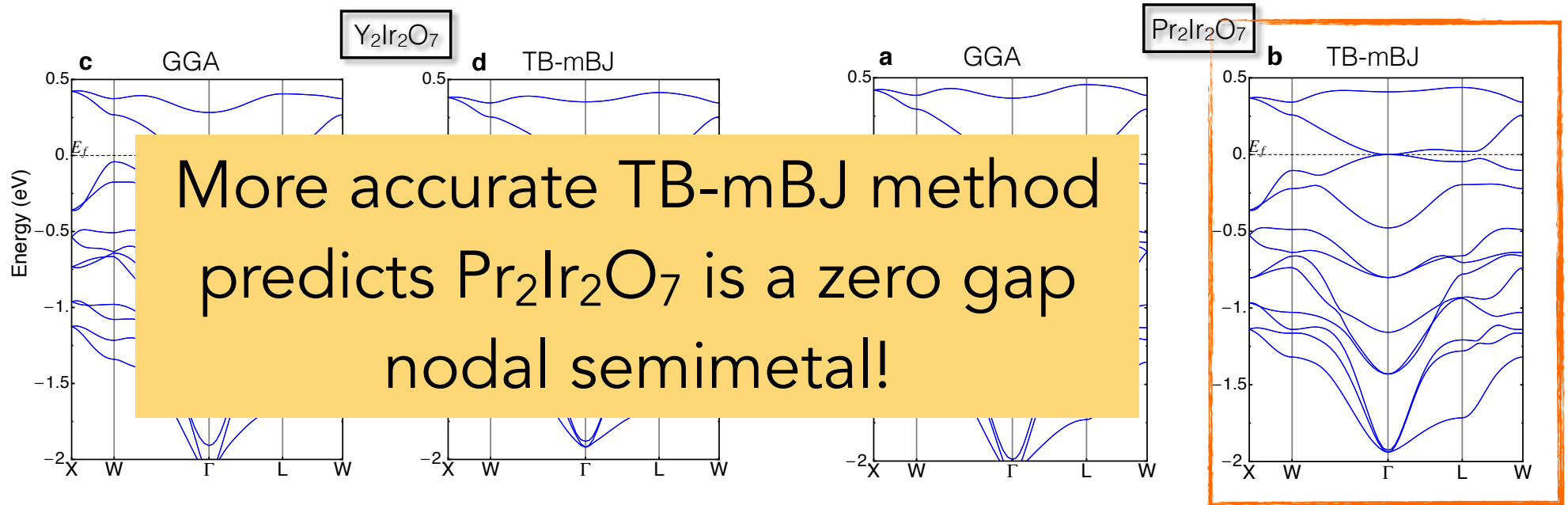


Band bending decreases with
increasing rare earth radius



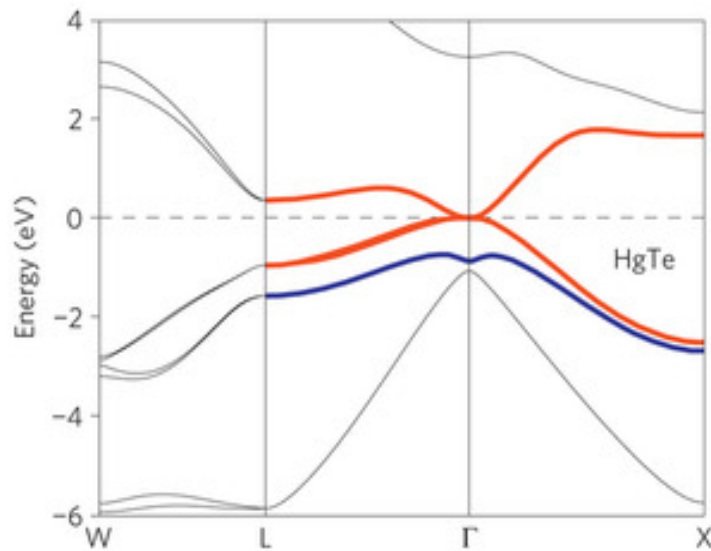
paramagnetic, GGA+SO, Wien2k

Electronic structure

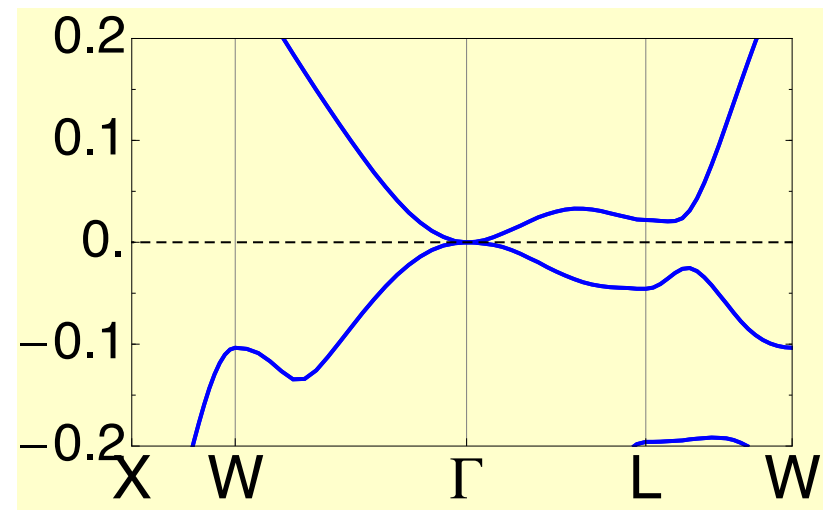


Band bending decreases with increasing rare earth radius

Quadratic band touching



HgTe

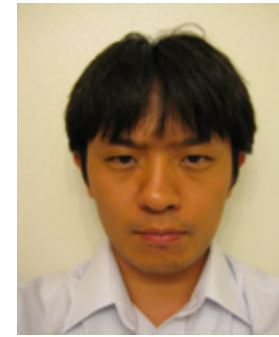


$\text{Pr}_2\text{Ir}_2\text{O}_7$

Mathematically identical band touching! But note energy scales



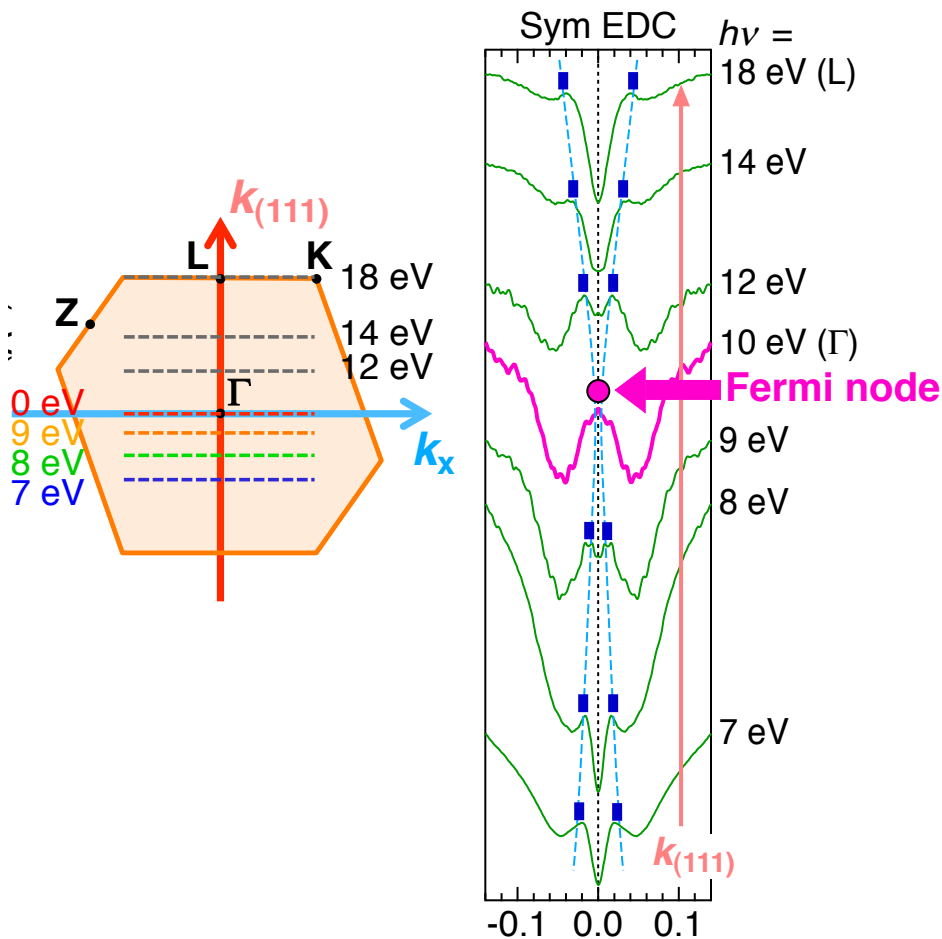
ARPES



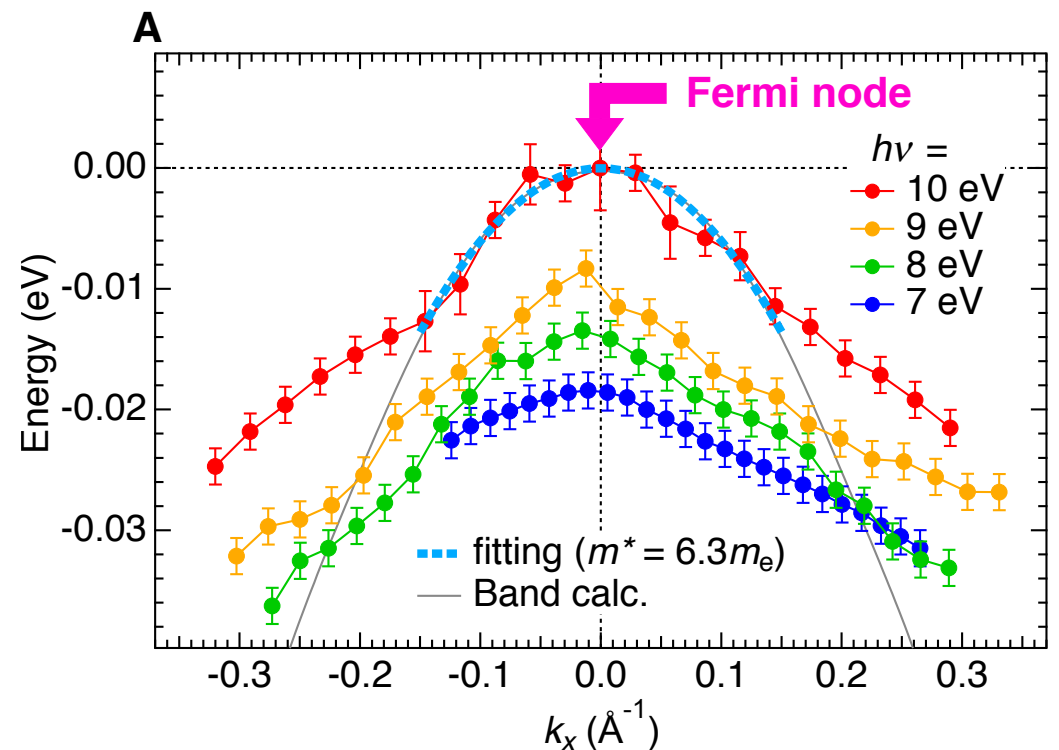
$\text{Pr}_2\text{Ir}_2\text{O}_7$ S. Nakatsuji

T. Kondo

S. Shin



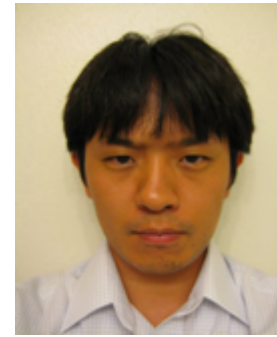
closest approach



Valence band approaches the Fermi energy at few meV resolution



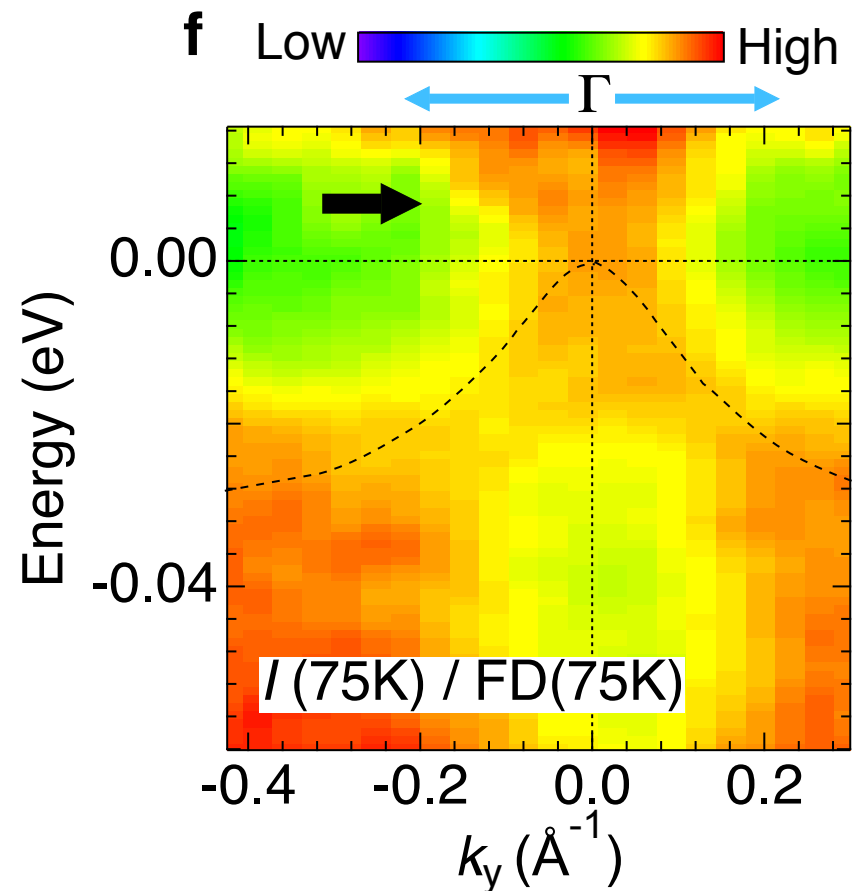
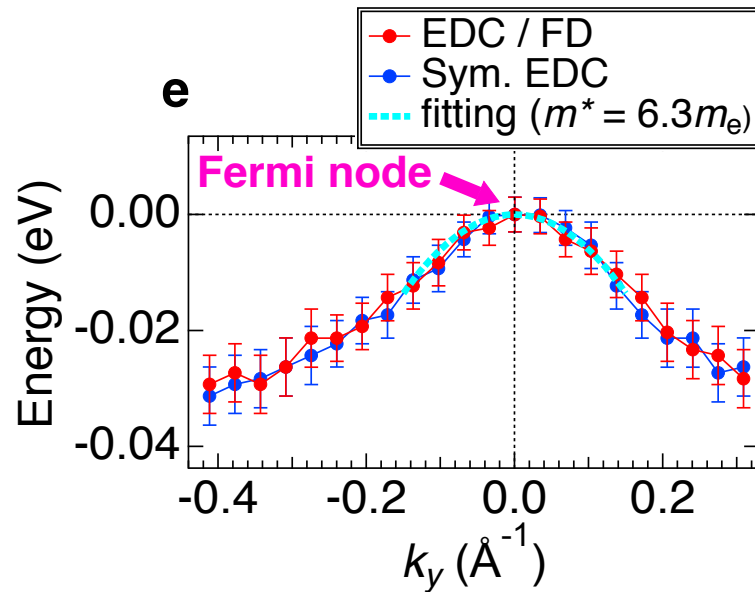
ARPES



$\text{Pr}_2\text{Ir}_2\text{O}_7$ S. Nakatsuji

T. Kondo

S. Shin





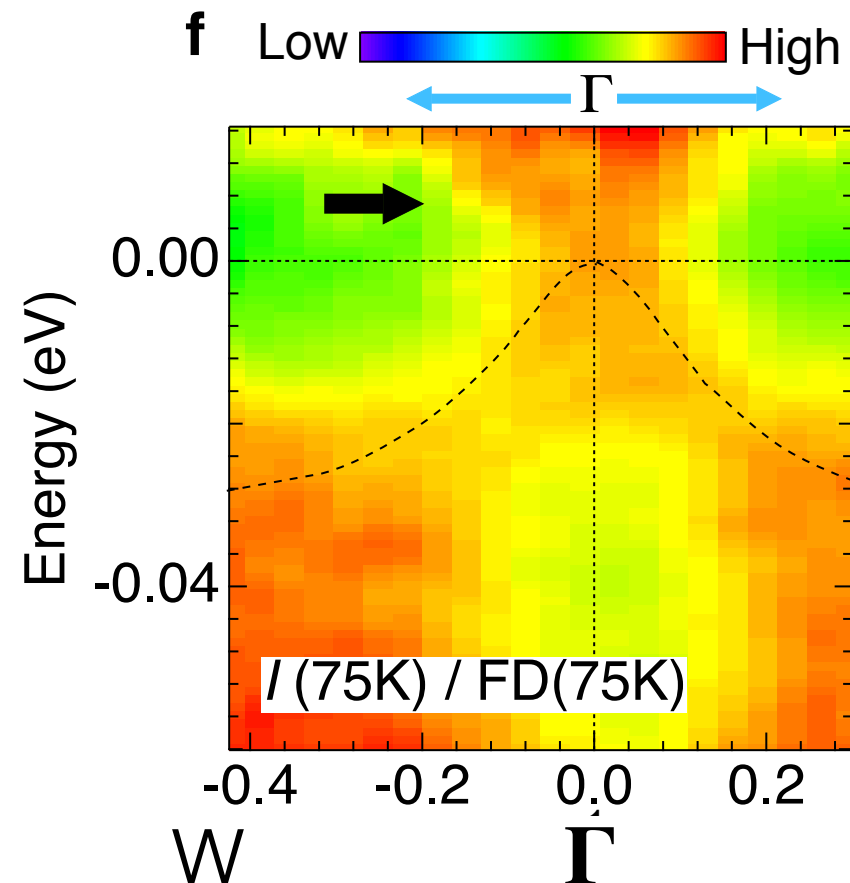
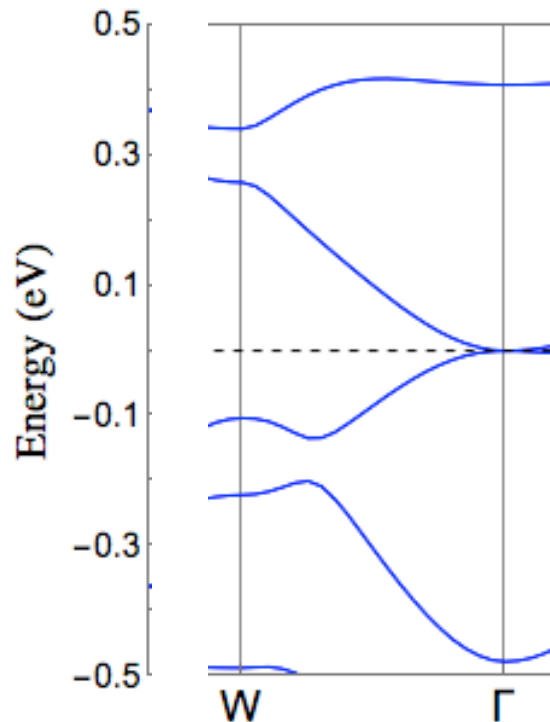
ARPES



$\text{Pr}_2\text{Ir}_2\text{O}_7$ S. Nakatsuji

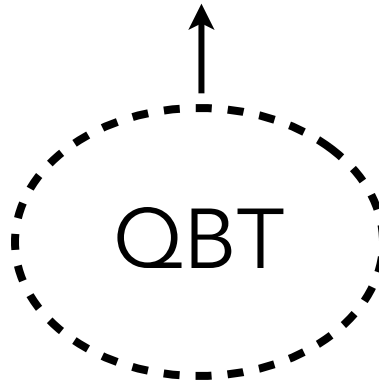
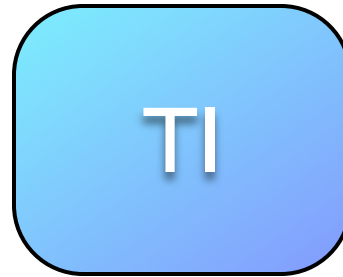
T. Kondo

S. Shin



overall bandwidth ~ 5 times reduced

$TI++$

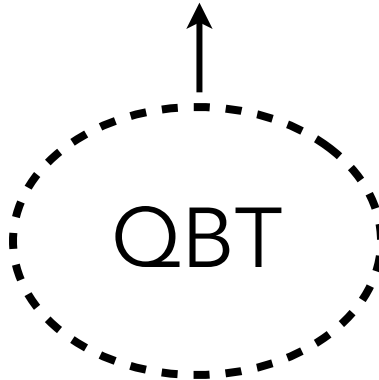


antiferromagnetism



Strong interactions

$TI++$

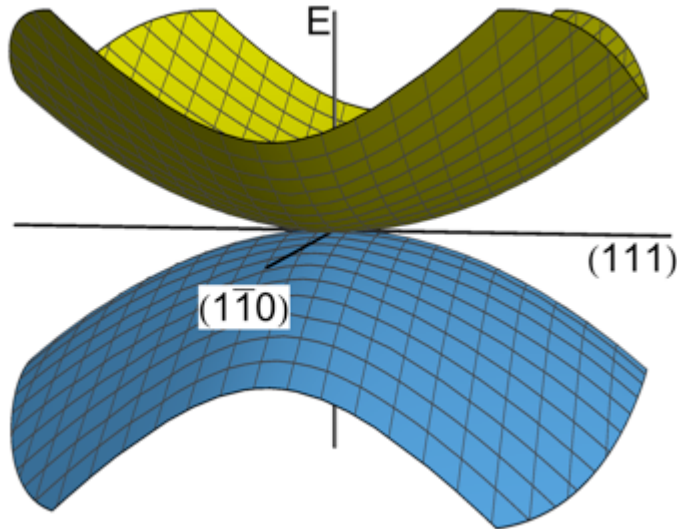


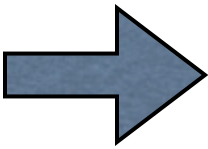
antiferromagnetism



Strong interactions

QBT to TI

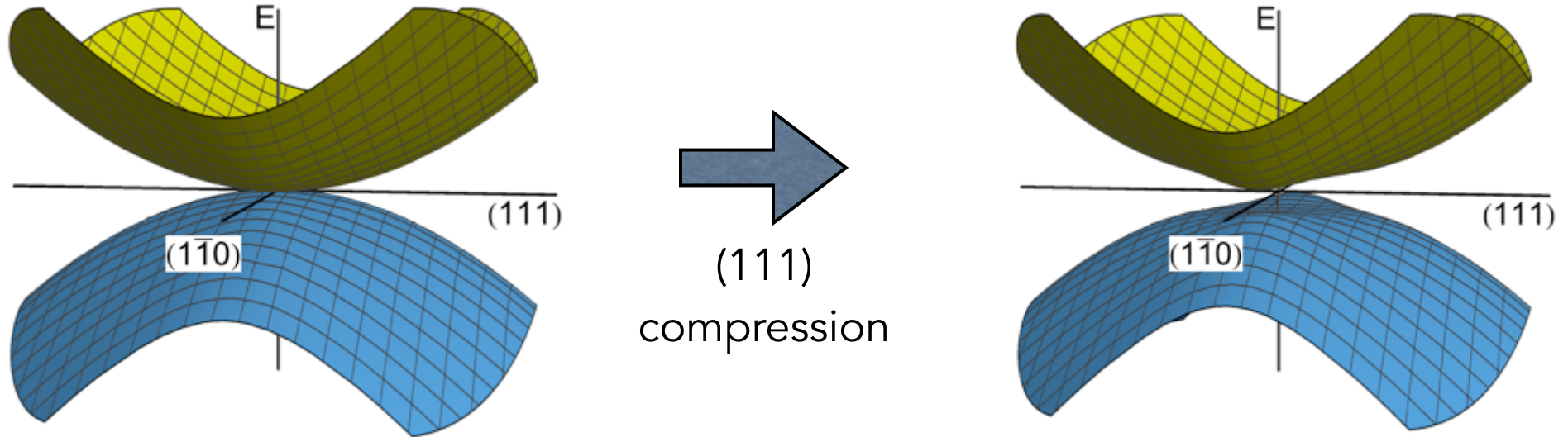



 (111)
 compression

???

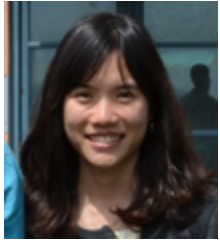
$$\begin{aligned}
 \mathcal{H}_0(k) = & \frac{d_a(k)}{2m} \Gamma^a + \frac{k^2}{2M_0} + \frac{d_4(k)\Gamma^4 + d_5(k)\Gamma^5}{2M_c} \\
 & + \varepsilon \Gamma^5
 \end{aligned}$$

QBT to TI



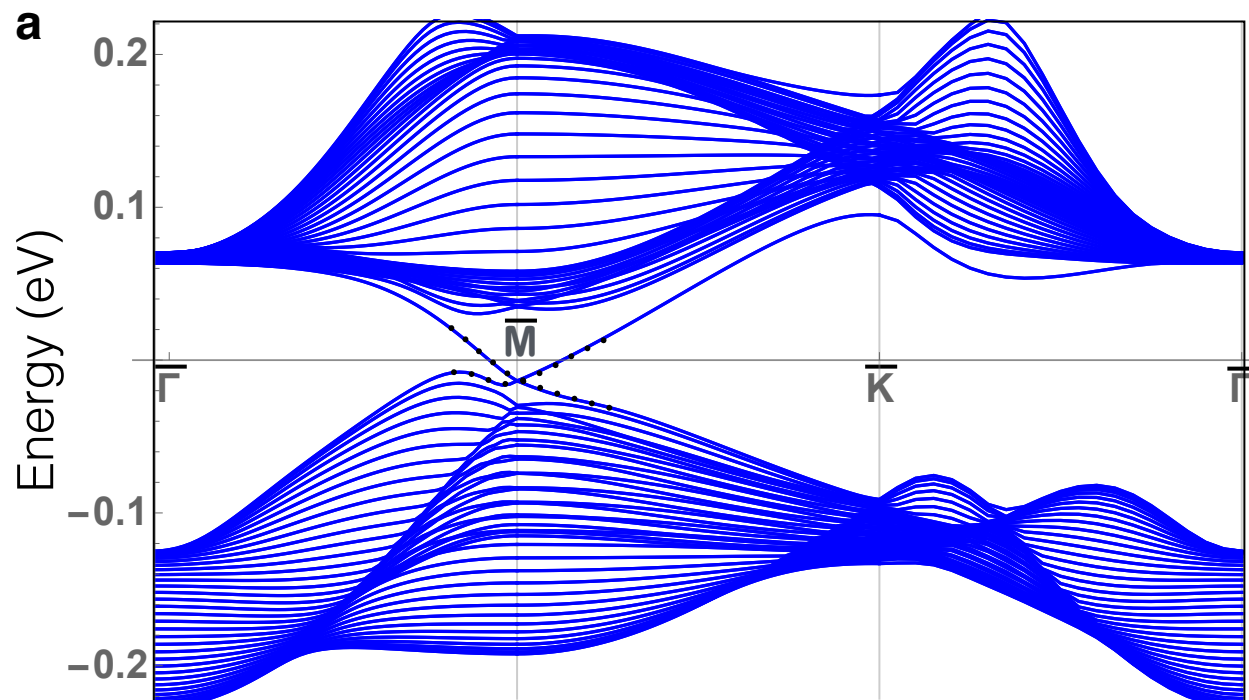
$$\mathcal{H}_0(k) = \frac{d_a(k)}{2m} \Gamma^a + \frac{k^2}{2M_0} + \frac{d_4(k)\Gamma^4 + d_5(k)\Gamma^5}{2M_c} + \varepsilon \Gamma^5$$

TI

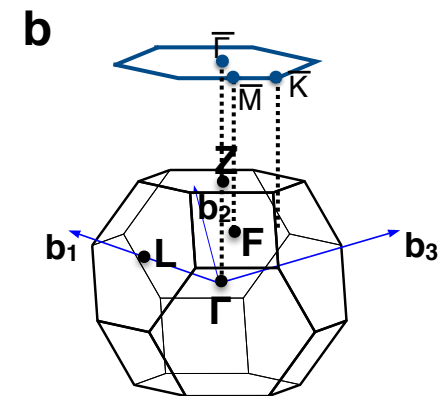


Topological Insulator

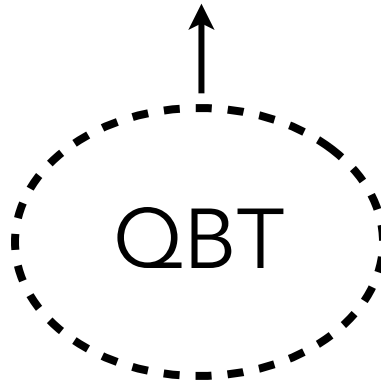
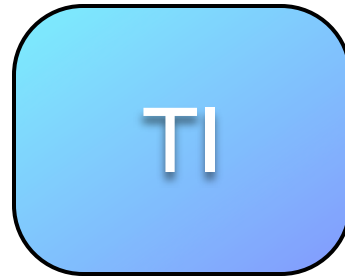
(111) compressive strain



3 surface Dirac cones



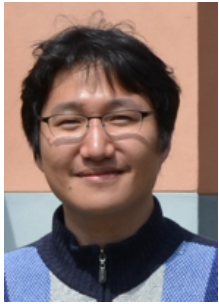
$TI++$



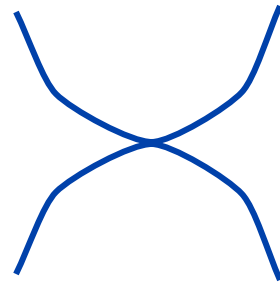
antiferromagnetism



Strong interactions



LAB phase



+ Coulomb = NFL

POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND DIELECTRICS

A. A. ABRIKOSOV and S. D. BENESLAVSKIĬ

L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

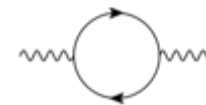
Zh. Eksp. Teor. Fiz. 59, 1280–1298 (October, 1970)

The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.

The role of the Coulomb interaction is considered for both types of spectra. In the case of a linear dispersion law a slowly varying (logarithmic) factor appears in the spectrum. In the case of a quadratic spectrum the effective interaction becomes strong for small momenta, and the concept of the one-particle spectrum turns out to be inapplicable. The behavior of the Green's functions is determined by similarity laws analogous to those obtained in field theory with strong coupling and in the neighborhood of a phase transition point of the second kind (scaling). Hence follow power laws for the electronic heat capacity and for the momentum distribution of the electrons.

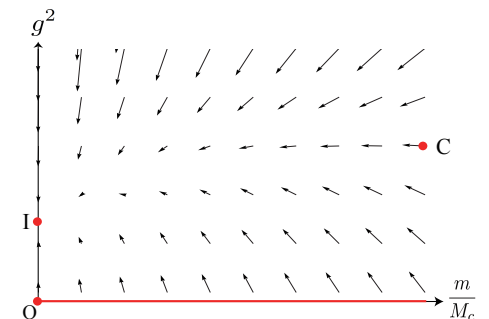
QFT

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\varphi + \hat{\mathcal{H}}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

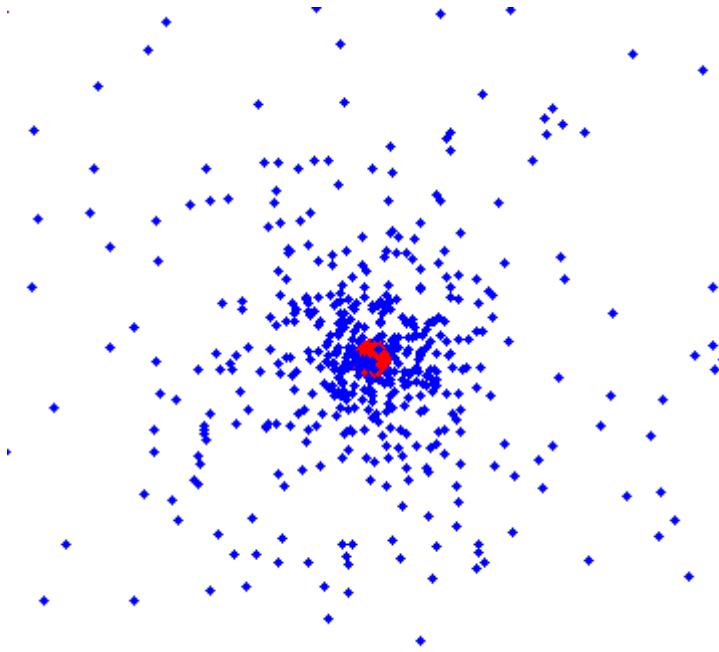


stable, NFL
fixed point

“Luttinger-Abrikosov-Beneslavskii” phase



Screening



© Brian Skinner

usual metal

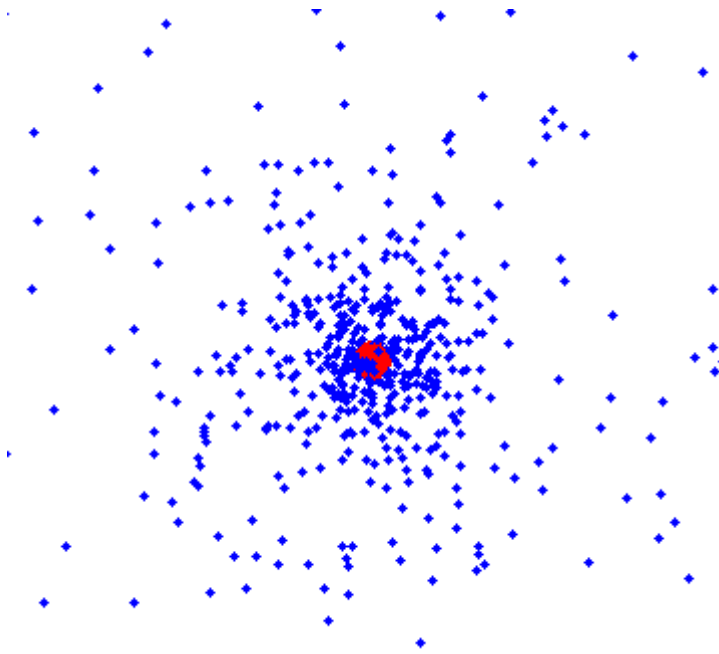
$$V(q) = \frac{1}{\frac{q^2}{4\pi e^2} + \frac{\partial n}{\partial \mu}}$$

$$V(r) = \frac{e^2}{r} \rightarrow \frac{e^2}{r} e^{-r/\lambda}$$

$$\lambda_{TF}^{-2} = 4\pi e^2 \frac{\partial n}{\partial \mu}$$

Screening

quadratic band touching



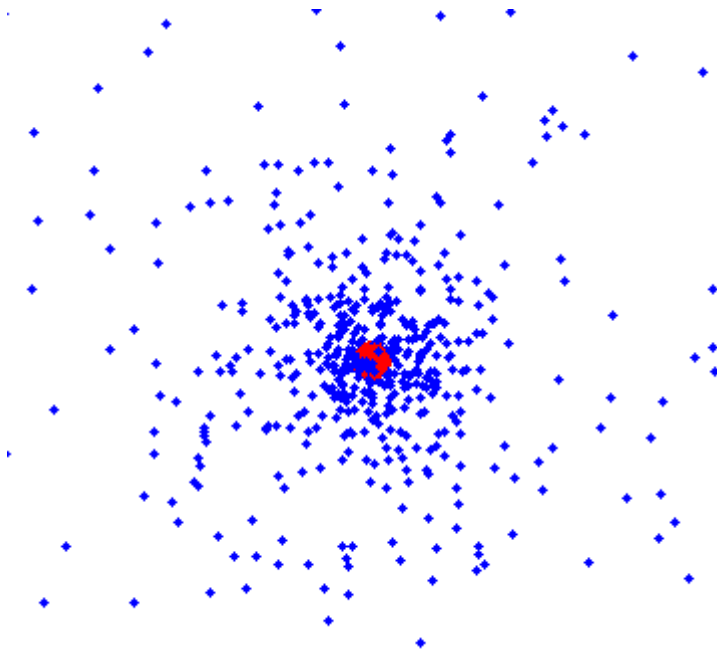
$$V(q) = \frac{1}{\frac{q^2}{4\pi e^2} + \frac{\partial n}{\partial \mu}}$$

$$\frac{\partial n}{\partial \mu} \sim \sqrt{\epsilon} \sim q$$

$$V(r) = \frac{e^2}{r} \rightarrow \frac{k}{r^2}$$

Screening

quadratic band touching



Quantum mechanics

$$V(q) = \frac{1}{\frac{q^2}{4\pi e^2} + \frac{\partial n}{\partial \mu}}$$

$$\frac{\partial n}{\partial \mu} \sim \sqrt{\epsilon} \sim q$$

$$V(r) = \frac{e^2}{r} \rightarrow \frac{k}{r^2}$$

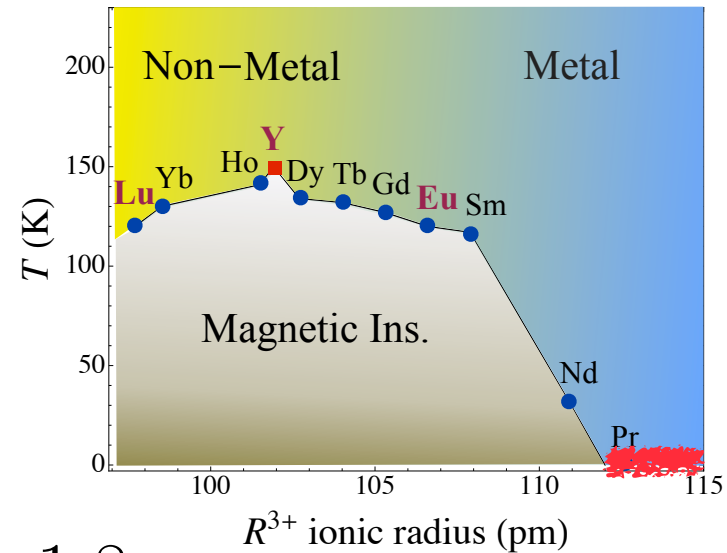
$$\left\{ -\frac{1}{2m} \left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + \frac{k}{r^2} \right\} \psi(r) = E\psi(r)$$

scale invariance!



NFL phase

Proper analysis:
large N or
epsilon
expansion



NFL

scaling,
e.g.

$$\omega \sim k^z \quad z \approx 1.8$$

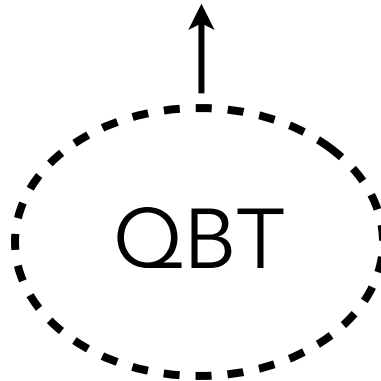
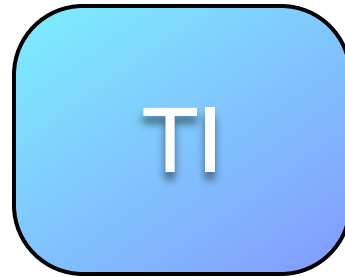
$$c_v \sim T^{d/z} \approx T^{1.7}$$

$$\sigma(\omega) \sim \omega^{1/z}$$

“Luttinger-Abrikoso-Beneslavskii”
phase

EG Moon, C. Xu, YB Kim, LB, PRL (2013)

$TI++$

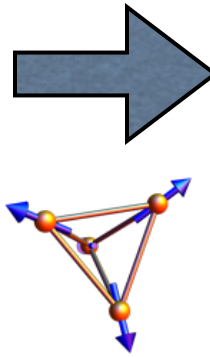
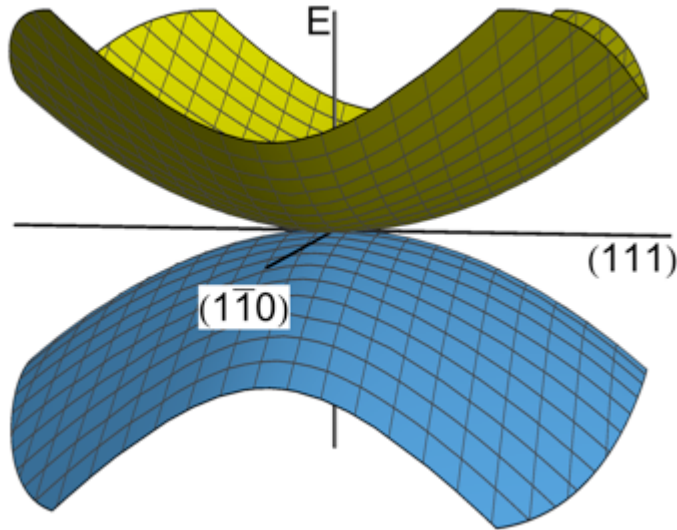


antiferromagnetism



Strong interactions

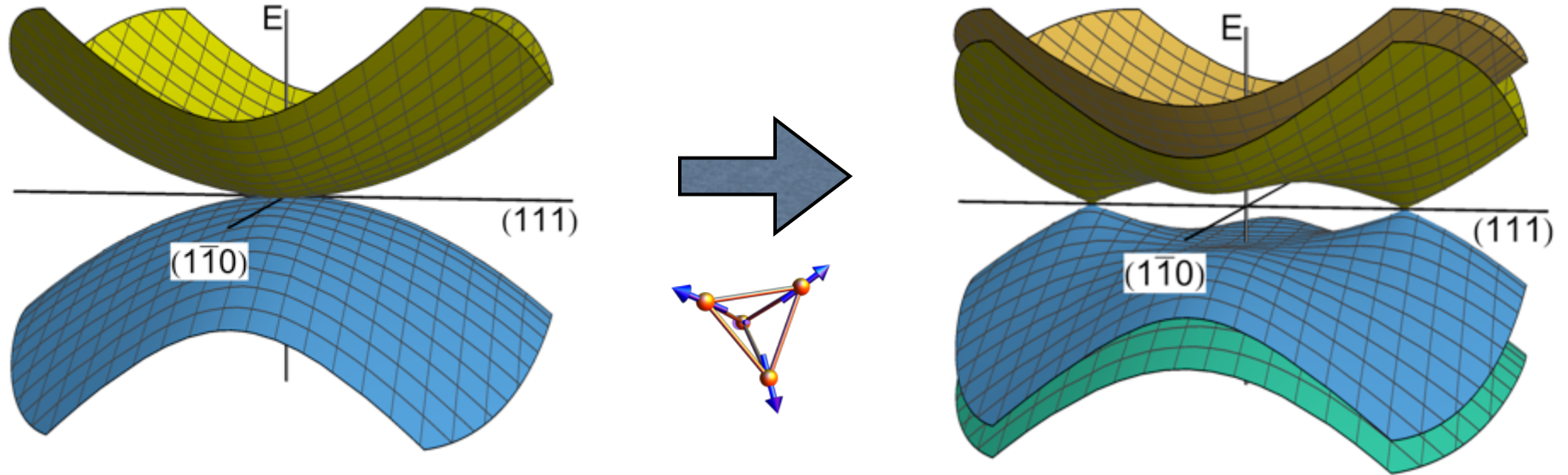
Weyl semimetal



???

$$\mathcal{H}_0(k) = \frac{d_a(k)}{2m} \Gamma^a + \frac{k^2}{2M_0} + \frac{d_4(k)\Gamma^4 + d_5(k)\Gamma^5}{2M_c} + i\phi \Gamma^4 \Gamma^5$$

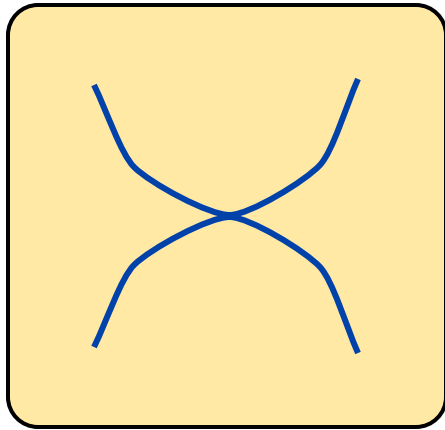
Weyl semimetal



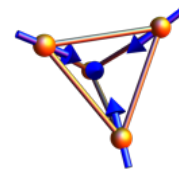
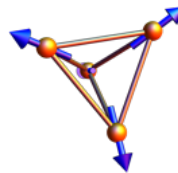
$$\mathcal{H}_0(k) = \frac{d_a(k)}{2m} \Gamma^a + \frac{k^2}{2M_0} + \frac{d_4(k)\Gamma^4 + d_5(k)\Gamma^5}{2M_c} + i\phi \Gamma^4 \Gamma^5$$

antiferromagnetic Weyl
semimetal

Field theory



+



= ??

ϕ

$$S = \int d\tau d^3x \left[\bar{\psi}_a (\partial_\tau + \hat{\mathcal{H}}_0) \psi_a + \frac{g}{\sqrt{N}} \phi \bar{\psi}_a \mathbf{M} \psi_a + \frac{r}{2} \phi^2 \right] \\ + \frac{ie}{\sqrt{N}} \varphi \bar{\psi}_a \psi_a + \frac{1}{2} (\nabla \varphi)^2$$

poor screening

Approach

large N_{fermion} expansion & RG

all-in-all-out
order
parameter

$$\Sigma_\phi = \text{[bubble diagram]} \sim |\mathbf{k}| |\ln c_1/c_2| f_\phi(\hat{\mathbf{k}}) + \sqrt{\omega_n} C \gg \overbrace{\mathbf{k}^2 + \omega_n^2}^{\mathcal{G}_{0;b}^{-1}}$$

Coulomb
interactions

$$\Sigma_\varphi = \text{[bubble diagram]} \sim |\mathbf{k}| |\ln c_1/c_2| f_\varphi(\hat{\mathbf{k}}) \gg \mathbf{k}^2_{\text{bare}}$$

$$\Sigma_f = \text{[wavy line bubble]} + \text{[dashed line bubble]}$$

$$\Xi_\phi = \text{[triangle with wavy line]} + \text{[triangle with dashed line]}$$

$$+ \text{[triangle with wavy line and dashed line]} + \text{[triangle with dashed line and wavy line]} + \text{[triangle with wavy line and dashed line]} + \text{[triangle with dashed line and wavy line]}$$

$$\Xi_\varphi = \text{[triangle with wavy line]} + \text{[triangle with dashed line]}$$

$$\propto 1/N_{\text{fermion}} \times \ln \Lambda$$

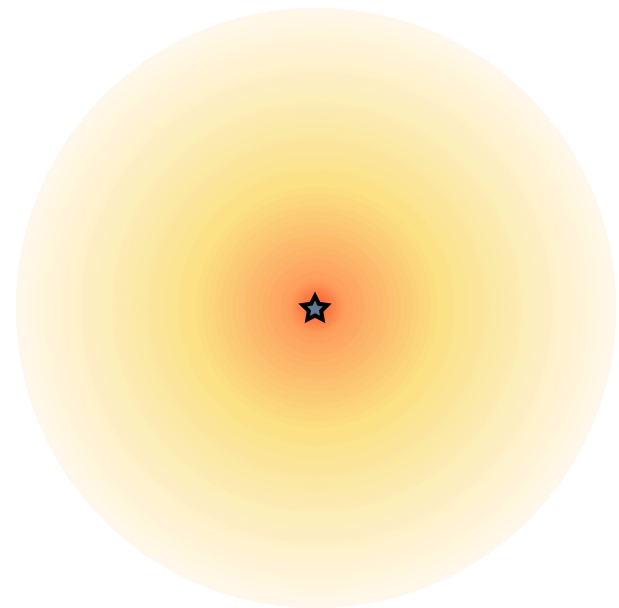
RG: calculate "only"
the coefficient of $\ln \Lambda$

Quantum criticality

- Stable fixed point with unusual exponents: extreme deviations from MFT

e.g. $\langle \phi \rangle \sim (r_c - r)^2$
(x logs)

wide QC
regime $z \approx 2$
 $\nu \approx 1$



Semimetal

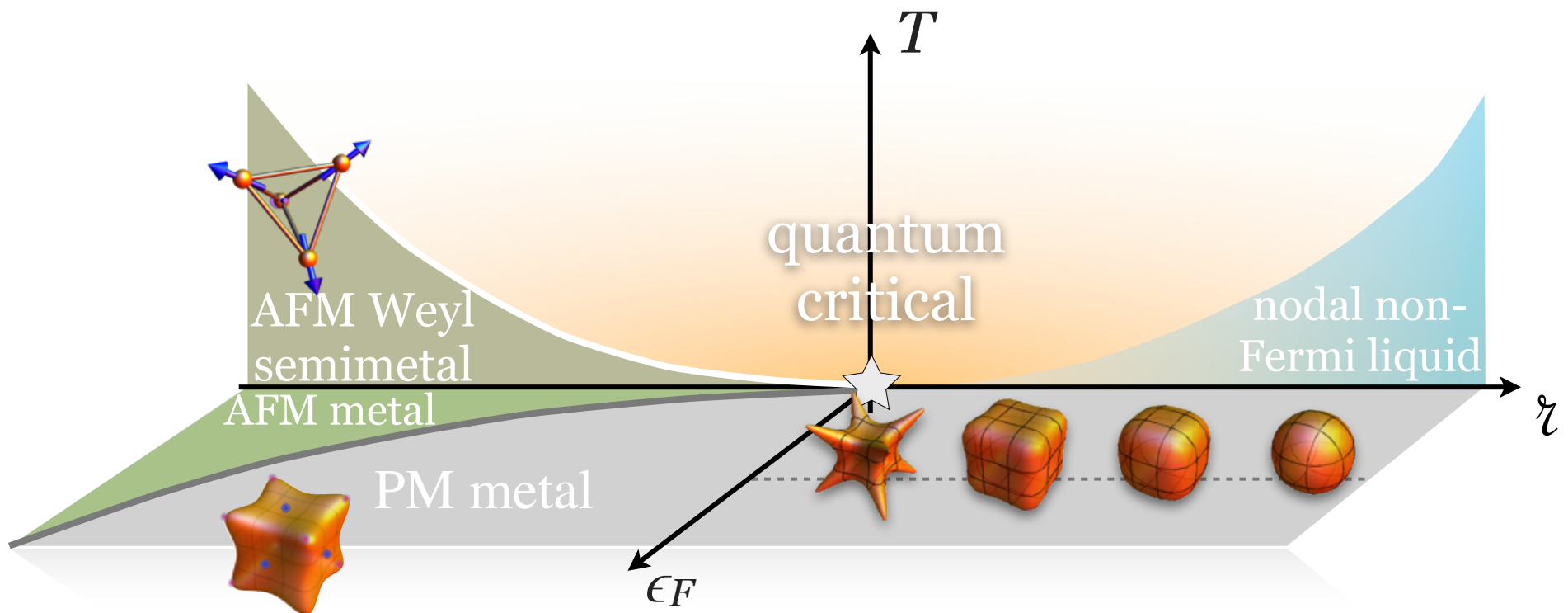
$$\chi^{-1} \sim q + \sqrt{|\omega|}$$

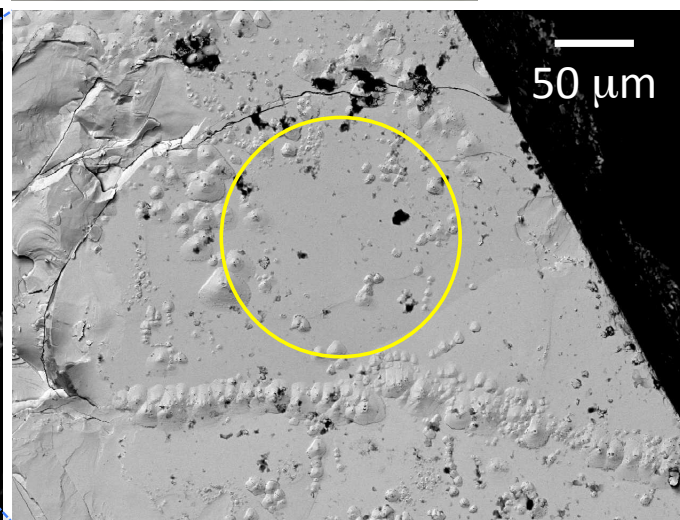
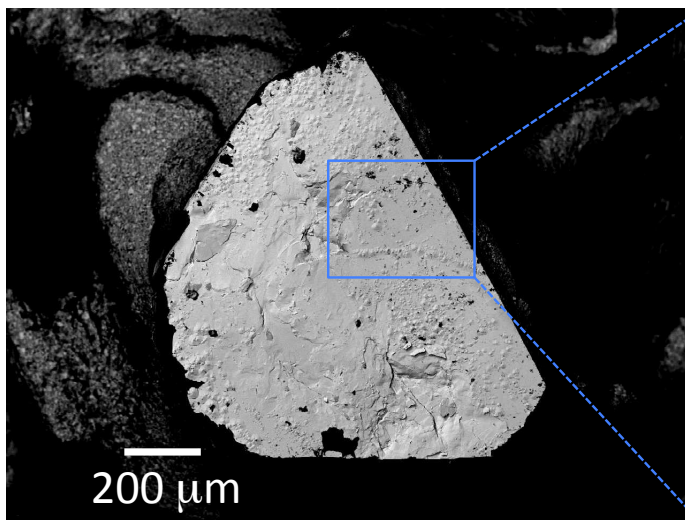
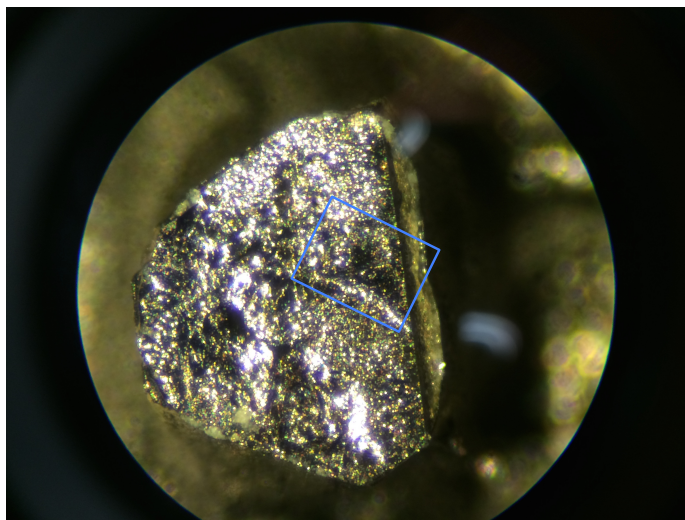
matched to electrons
electrons scatter strongly



Quantum criticality

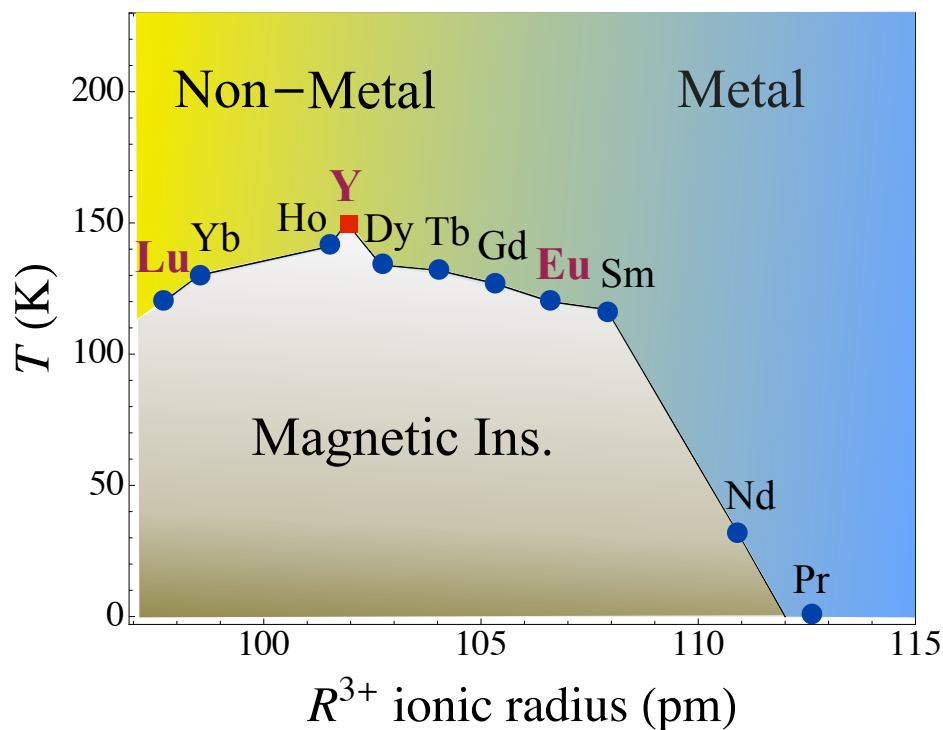
- QBT leads to extremely non-classical QCP





Pyrochlore iridates

- Weyl semimetal?

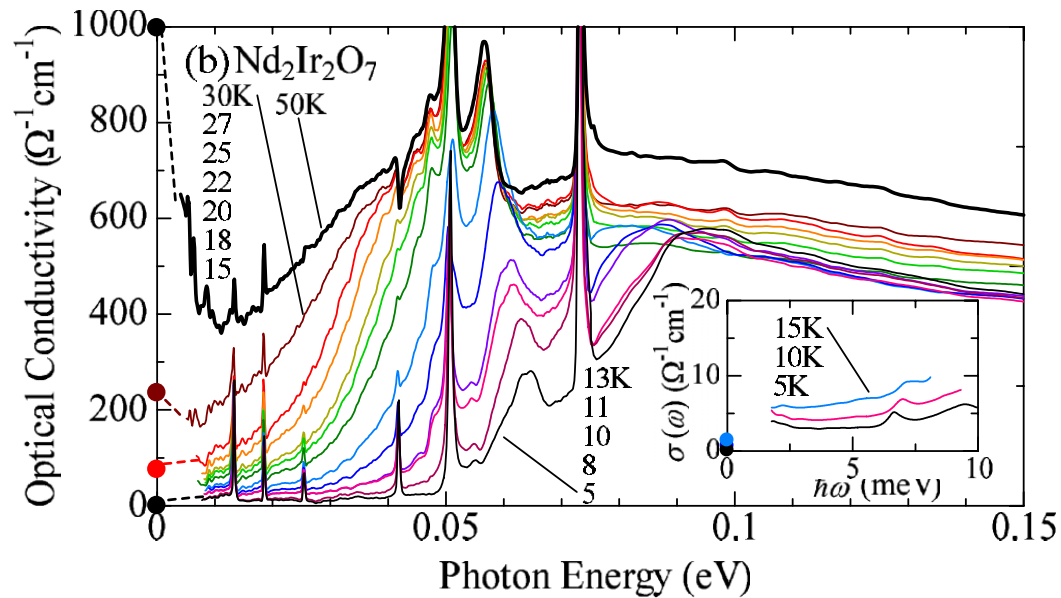


Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCOMP 2013

Best candidate
should be
 $\text{Nd}_2\text{Ir}_2\text{O}_7$

Weyl not?

K. Ueda *et al*, 2012

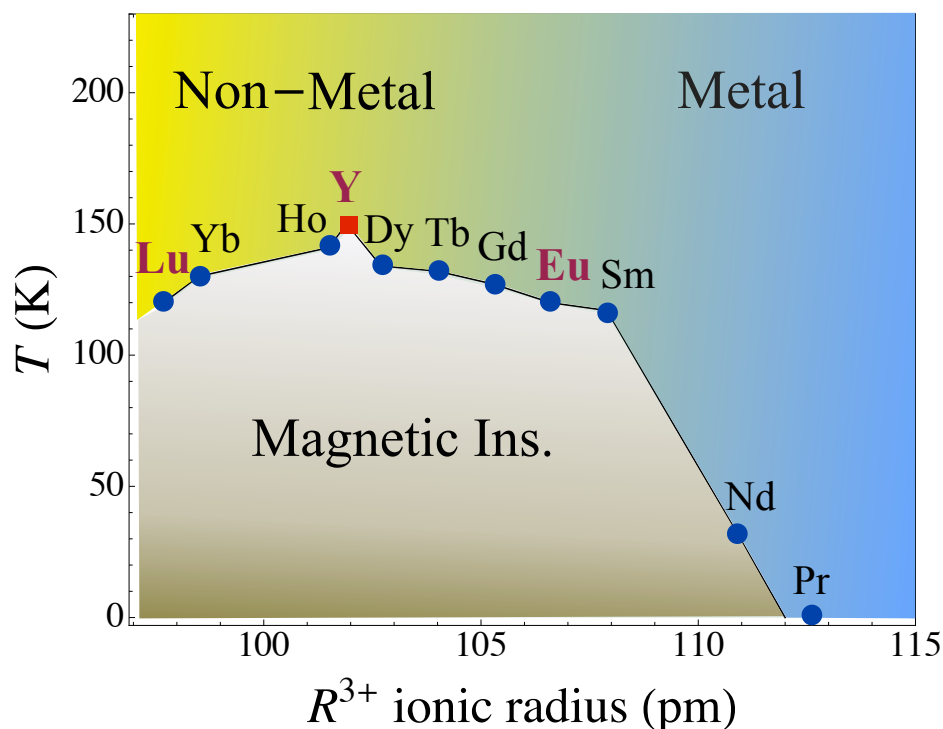


Clear charge
gap $\sim 45\text{meV}$



Pyrochlore iridates

- Continuous magnetic/metal-insulator transitions

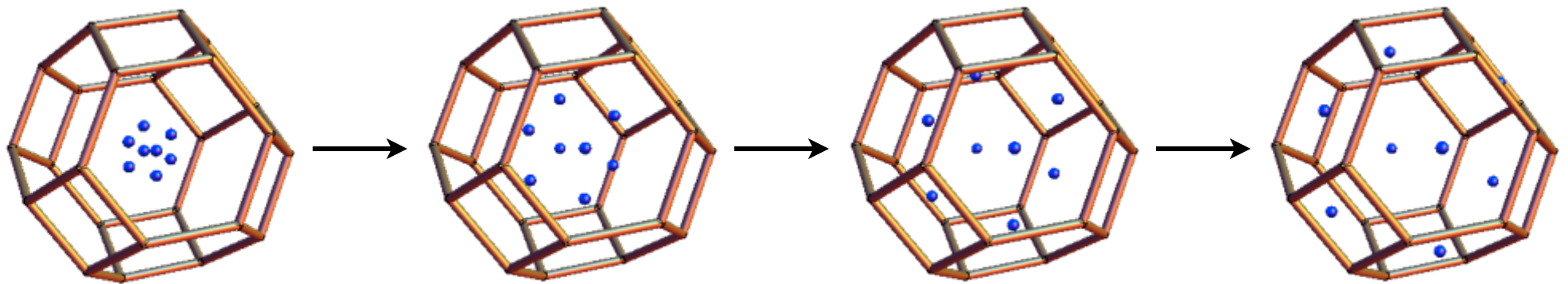
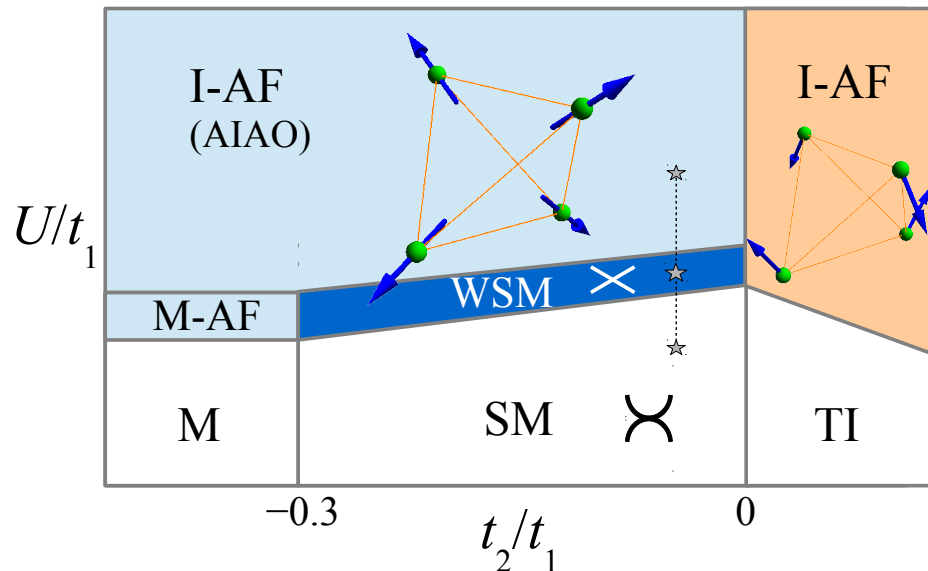


R^{3+} ionic radius (pm)
Yanagashima+Maeno, JPSJ 2001
K. Matsuhira et al, JPSJ 2011
W. Witczak-Krempa et al, ARCOMP 2013

Puzzling things:

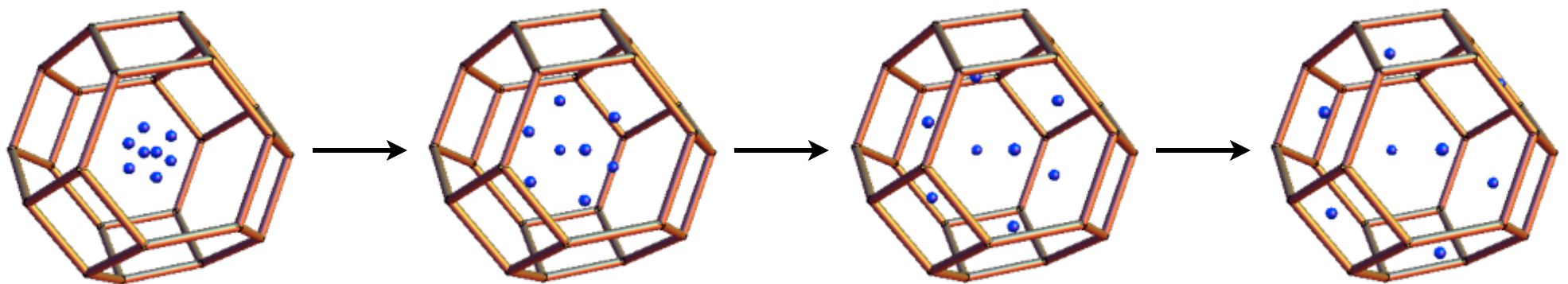
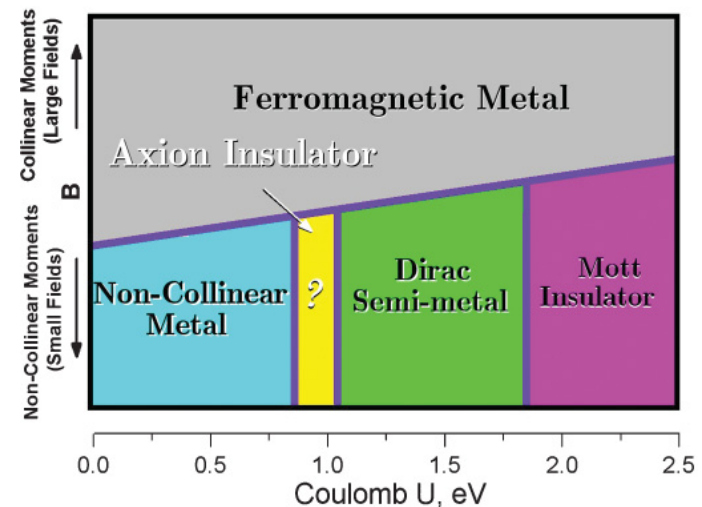
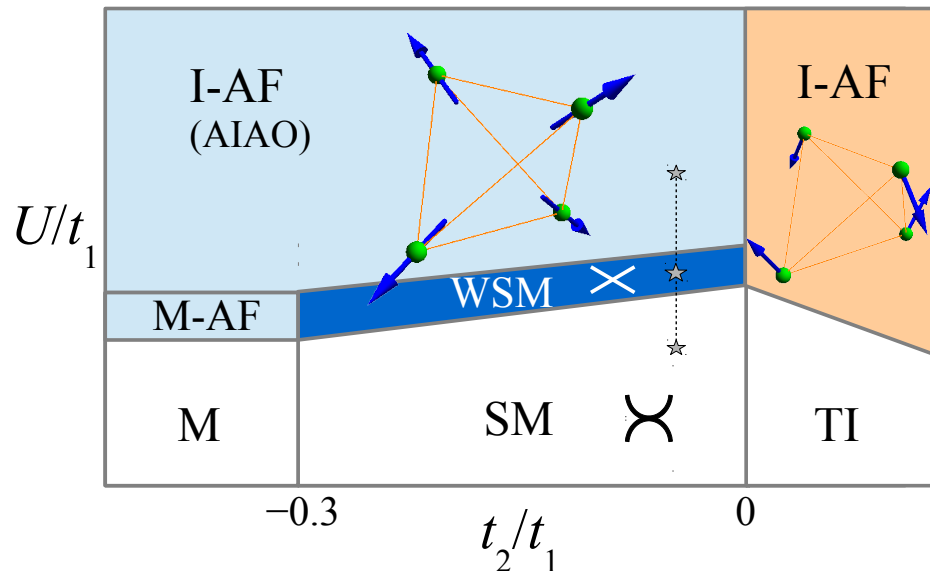
- Already a gap for weakest ordering?
- $T_c = 30$ K, gap > 500 K
- Why such low scales when $U \sim t \sim 10^4$ K?

Weyl not?



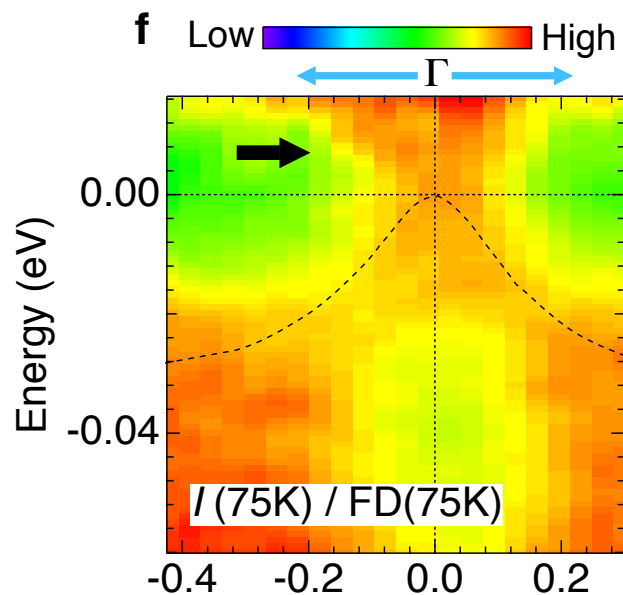
Weyl points move to zone boundary and annihilate with increasing order?

Weyl not?

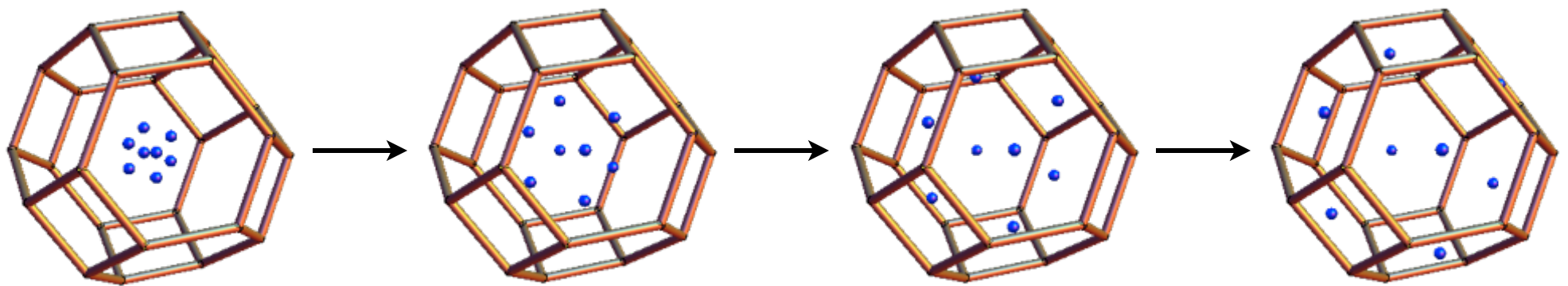


Weyl points move to zone boundary and annihilate with increasing order?

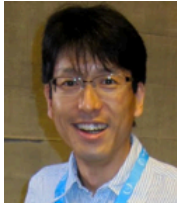
Weyl not?



maybe bandwidth reduction
makes this possible



Weyl points move to zone boundary and annihilate with increasing order?



unpublished

Metal-insulator transition

very sensitive to stoichiometry



Metal-insulator transition

very sensitive to stoichiometry

field *close to* (100) kills
the insulating state

Metal-Insulator Transition

- Why only along (100)?
- Why such a small field?
- What is the nature of the metallic state?

Metal-Insulator Transition

- Why only along (100)?
- Why such a small field?
- What is the nature of the metallic state?

Due to Nd
moments

Nd physics

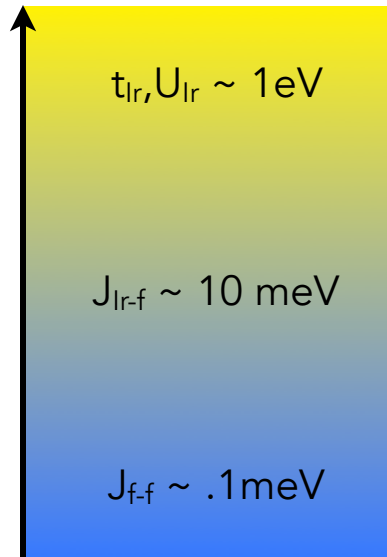
- Nd has large moment: $\sim 2.3\mu_B$
- Similar to “spin ice” Ising moments?

Nd physics

- In detail it is quite distinct from spin ice. Simple Ising moments would have abrupt features for all orientations.

Physics

- Kondo lattice $H = H_{Ir} + H_K + H_B$



$$H_K = \sum_{ia} \vec{S}_i \cdot J_{ia} \cdot \vec{\tau}_a$$

$$H_B = - \sum_i \vec{B} \cdot g_i \cdot \vec{S}_i - \sum_a \vec{B} \cdot g_a \cdot \vec{\tau}_a$$

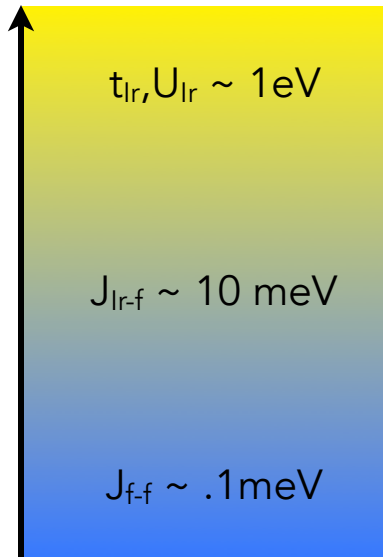
Ising-like
moments

$$(g_a)_{\mu\nu} = g \hat{e}_\mu \delta_{\nu,3}$$

Octupolar-dipolar doublet:
Y.P. Huang *et al*

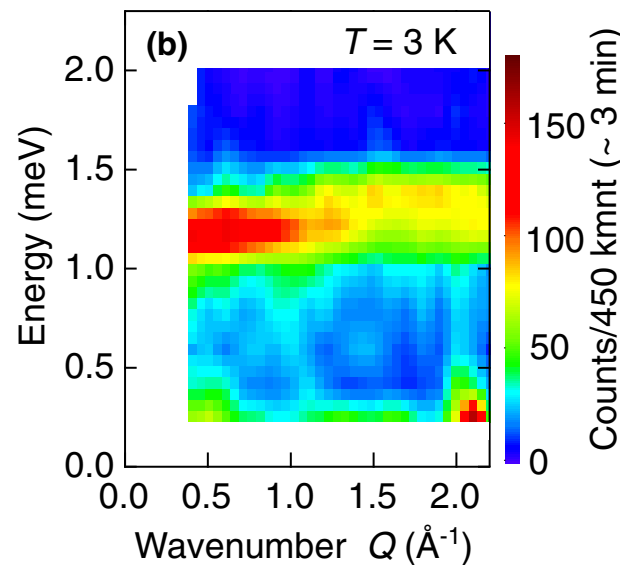
Physics

- Kondo lattice $H = H_{Ir} + H_K + H_B$



Ising-like
moments

$$H_K = \sum_{ia} \langle \vec{S}_i \rangle \cdot \mathbf{J}_{ia} \cdot \vec{\tau}_a \quad \text{gives effective field}$$



Tomiyasu et al, 2012

Nd Zeeman
splitting $\sim 1.2\text{meV}$

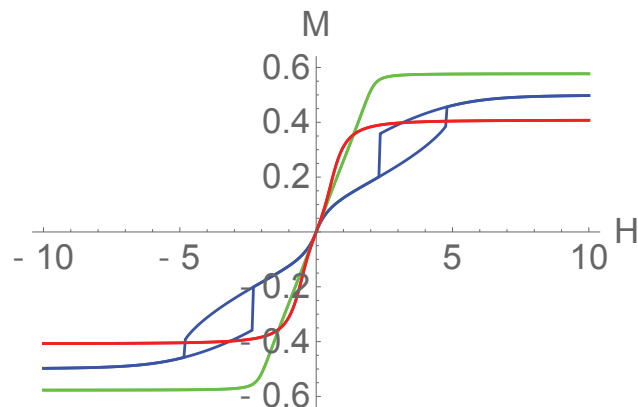
J probably larger
as Ir ordered
moment is small

Magnetization

- Since magnetization is due to Nd, can integrate out Ir to obtain effective spin model

$$H_{Nd} = \sum_{\langle ij \rangle} J_z \tau_i^z \tau_j^z + J_y \tau_i^y \tau_j^y + J_x \tau_i^x \tau_j^x + J_{xz} (\tau_i^x \tau_j^z + \tau_i^z \tau_j^x) - A_H \sum_i \tau_i^z - g \sum_i \tau_i^z (\hat{n}_i \cdot \bar{H})$$

RKKY
exchange
+Zeeman fields

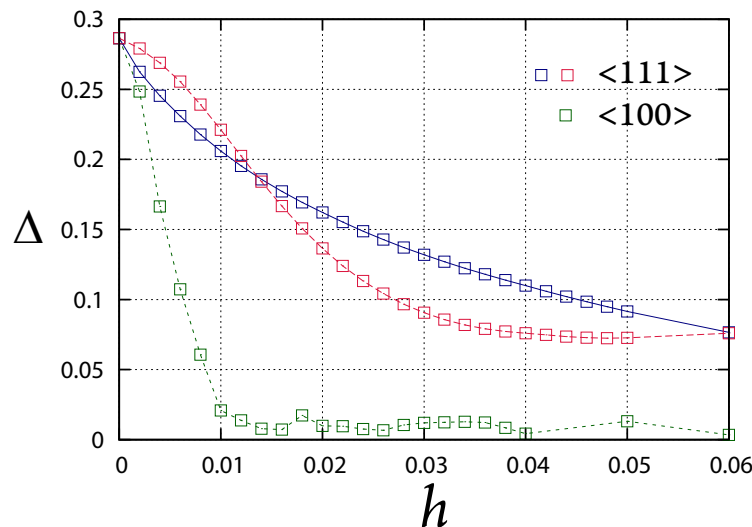


Explains M curves *if*

- Non-Ising terms substantial
- RKKY interactions are ferromagnetic

Anisotropy

- Intrinsic strong sensitivity of Nd to field direction is transferred to Ir



Hartree-Fock on
Hubbard+Kondo lattice
model

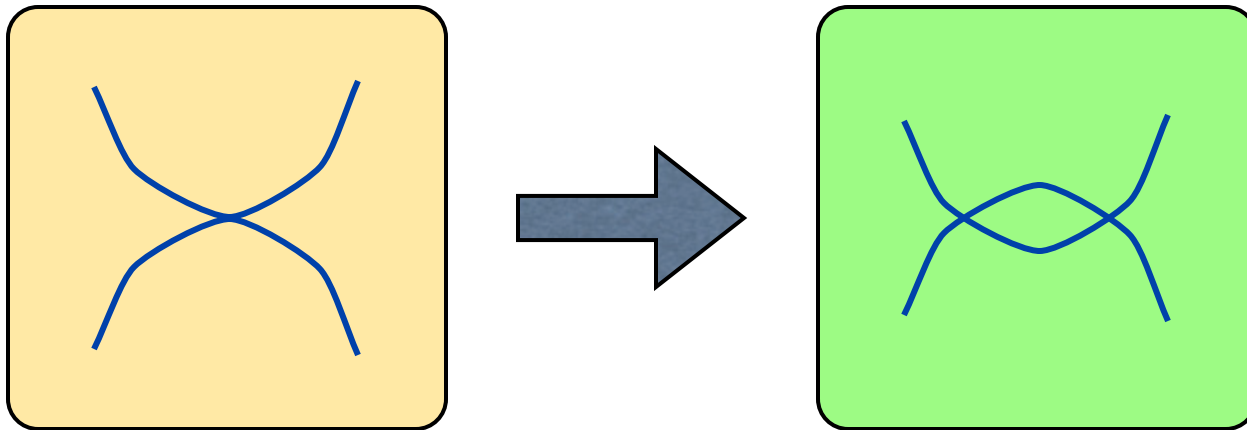
Kondo coupling completely
overwhelms direct Zeeman field

Metal-Insulator Transition

- Why only along (100)?
- Why such a small field?
- What is the nature of the metallic state?

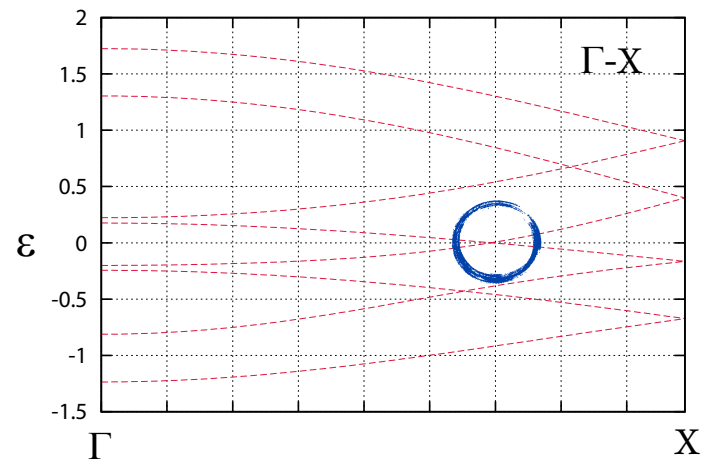
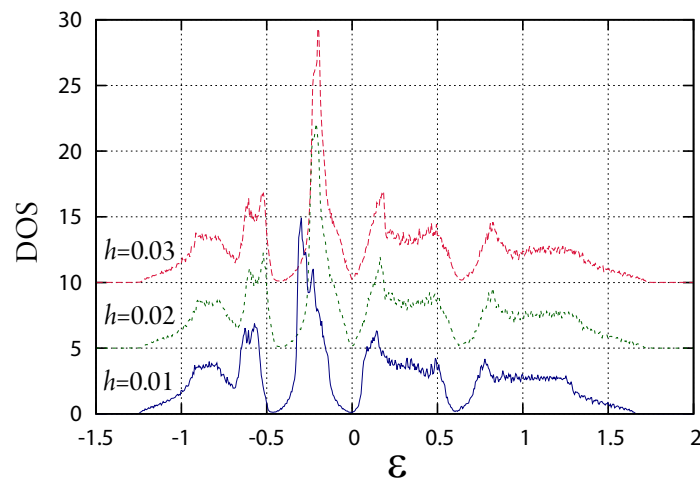
Weyl?

- Guess: field completely suppresses AIAO Ising order
- Like quadratic band touching with uniform field?



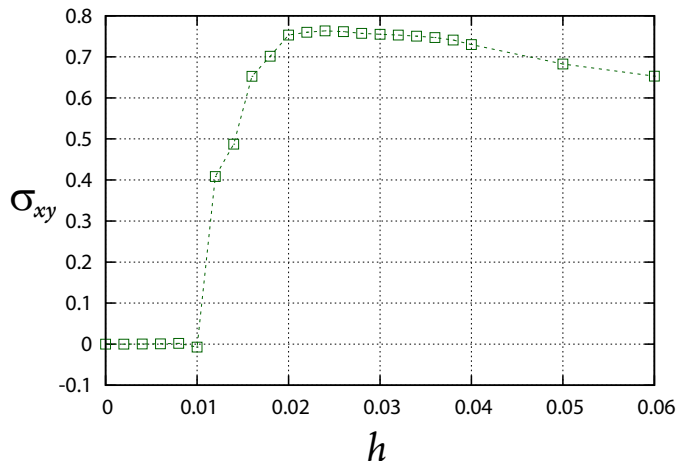
Weyl?

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Weyl?

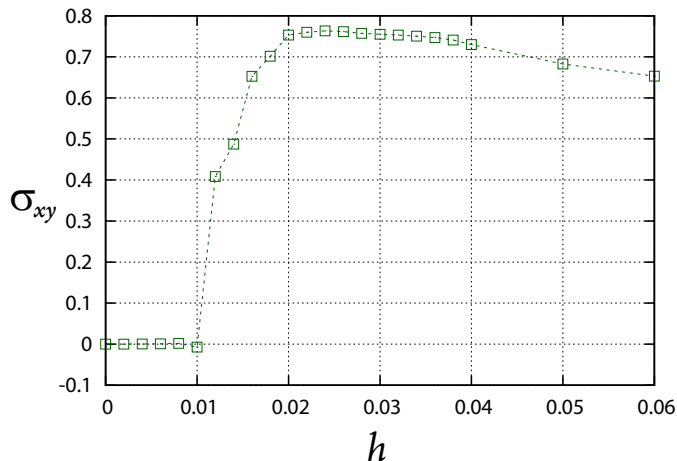
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Simple model gives
large Hall conductivity

Weyl?

- Guess: field completely suppresses AIAO Ising order
- Like quadratic band touching with uniform field?



Simple model gives
large Hall conductivity
(too large)

Conclusions

- The paramagnet electronic structure of $A_2Ir_2O_7$ may realize a strongly correlated analog of HgTe
- In theory, this supports a topological insulators, Weyl semimetals, and quantum criticality
- In practice, rare earth spins crucially affect low energy physics and transport.
- Good side: makes it possible to *control* electronic state